ELEMENTARY DYNAMO DESIGN

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WITH NUMERICAL EXAMPLES

BY

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WITH 128 DIAGRAMS

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PREFACE

The aim of this volume is to explain, by means of numerical examples, the methods and calculations necessary for the design of dynamo-electric machinery. Controversial points as to the nicety of design are avoided as belonging to a more advanced study of the subject than is here intended, but the general effect of different modifications are noted as occasion arises.

Whilst no endeavour has been made to treat of the theory of electricity and magnetism, introductory chapters deal with points that are most intimately connected with dynamo design in such a way as will, it is hoped, enable those who approach the subject from the practical side, and without any deep theoretical training, intelligently to follow the reasoning in the succeeding chapters.

A system of mixed units is used throughout the book: the dimensions of all parts of the machines are given in inches, whilst all the magnetic calculations are worked out in centimeters. It is necessary that the finished results should be put into the shops in feet and inches, but all scientific works and original papers dealing with magnetism invariably base their calculations on the centimeter as the unit of length. If a design be worked out entirely in metric units, and these be afterwards converted into inches, many of the dimensions will come out with undesirably small fractions; to round them off will necessitate revision of all the calculations. The system used here has been adopted as the least inconvenient of the possible compromises between working entirely in metric or entirely in British measure. That such a clumsy and unscientific expedient should be necessary is a strong indictment against the system of weights and measures in use in the United Kingdom.

In the appendix there will be found a table of constants useful in converting from metric to British units, and there is also given a sample table of the properties of copper conductors.

At the beginning of the book is given a list of the abbreviations and symbols used throughout the work.

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LIST OF ABBREVIATIONS AND SYMBOLS IN COMMON USE

E.M.F. = Electromotive force.
C.C. = Continuous current.
A.C. = Alternating current.
C.G.S. = Centimeter-gramme-second system of units.
S.W.G. = Standard wire gauge.

H.P. = Horse-power.
B.H.P. = Brake horse-power.
E.H.P. = Electrical horse-power.
A.T. = Ampere-turn.
R.M.S. = Root mean square value of alternating current or E.M.F.
P.F. = Power factor.

E = Electromotive force in volts.
C = Current measured in amperes.
R = Revolutions per minute.
ω = Resistance measured in ohms.
H = Magnetic force measured in C.G.S. units.
B = Magnetic induction = Number of lines of magnetic force per square centimeter.
N = Total number of lines of magnetic force from one pole.
μ = Magnetic permeability = $\frac{B}{H}$.

ELEMENTARY DYNAMO DESIGN

CHAPTER I

INTRODUCTORY

§ 1. **Electro-Magnetic Induction.**—All types of dynamos and motors, whether for use with continuous or alternating currents, are included under the term dynamo-electric machinery.

The object of dynamo-electric machinery is to convert mechanical into electrical energy or, being supplied with electrical energy, to convert it into mechanical.

To convert mechanical into electrical energy is the function of a generator, which, driven by a prime mover, gives out current at its terminals. To convert electrical energy into mechanical is the function of a motor which, being supplied with current, gives out mechanical energy at the pulley or coupling.

The principle underlying all these machines is that of electro-magnetic induction, namely, that whenever a conductor is moved in a magnetic field, so as to cut lines of magnetic force, an E.M.F. is induced in the conductor.

Magnets used to produce the necessary magnetic field may be permanent magnets or electro-magnets. Permanent magnets are pieces of hard steel which, having once been magnetised, retain a large amount of magnetism. Electro-magnets are also steel or iron; they carry windings of copper wire through which an electric current is passed.

The effect of the current is to change for the time being the steel round which it is flowing into a magnet. This effect, however, dies out to a great extent when the current is interrupted.

§ 2. **The Magnetic Field.**—The space surrounding a magnet and under its influence is known as the magnetic field. Throughout the magnetic field the presence of a magnetic force can be detected, a small magnet sets itself in a particular direction owing to the attraction and repulsion of the larger magnet ; small pieces of iron are themselves magnetised, and either set themselves in special directions, or if free to move, approach the magnet and adhere to it.

At all points of the field the force of attraction has a definite direction, and lines drawn so as to indicate this direction at every point are known as lines of magnetic force.

A conception of lines of magnetic force plotted throughout the magnetic field is most useful, and a thorough grasp of it is indispensable to anyone undertaking the study of dynamo-electric machinery.

The general direction of the lines of force can easily be shown experimentally. If a magnet be placed under a sheet of stiff paper, and iron filings scattered over the surface of the paper, they will show a tendency to set themselves along distinct lines. This tendency may be assisted by gently tapping the paper, when the filings will be seen to set themselves along distinct curves all over the surface. These curves are lines of magnetic force ; they show at any point the direction along which the magnetic force at that point acts.

Lines of force may be plotted more accurately by using a small compass needle. A magnet being placed on a sheet of paper, and the compass needle brought near it, the needle sets itself in a definite direction. The north and south poles of the compass needle are marked by a dot on the paper. Assume that the north pole was nearest the magnet. The compass needle is now moved so that its north pole coincides with the point formerly occupied by the south one. The south pole is again marked by a

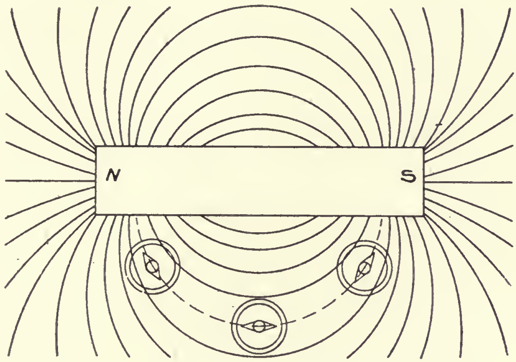


FIG. 1.

dot, the north pole again brought up to this point, and so on, marking each fresh position of the south pole by a new dot. Proceeding in this way, and joining all the dots so obtained, a curve will be placed on the paper which, starting from the neighbourhood of the south pole of the magnet, will gradually curve round to the north end (see Fig. 1). Any number of curves can be so drawn until the whole field is plotted out into lines of magnetic force. Every line showing at all points the direction in which a small magnet will set itself, therefore indicates the direction in which the magnetic force due to the large magnet acts.

In the experiments above described the lines have been drawn in one plane only, but they must of course be thought of as spreading in all planes round the magnet and filling the whole of the region of space under its influence. Each of the lines of magnetic force drawn in this way will emerge from the magnet near one end, and enter it again in the neighbourhood of the other. The process cannot be continued so as to trace the path of the line through the substance of the magnet itself, but if a hollow coil of wire carrying a current is substituted for the magnet the same

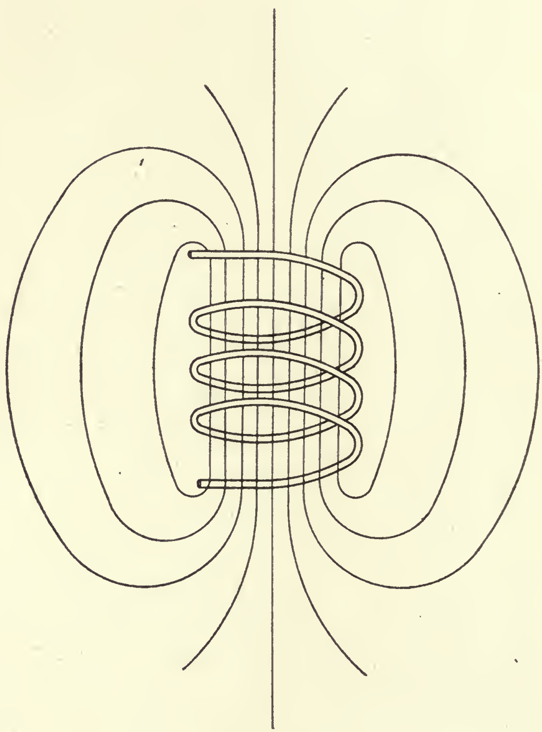


FIG. 2.

effects are found in the neighbourhood of the coil, and a diagram of lines of magnetic force can be plotted as described above. In this case, however, the path of the lines can be continued through the centre of the solenoid, and it is then found that each line returns on itself and forms a closed curve (Fig. 2). The lines curving from a magnet must, in the same way, be thought of as being completed through the substance of the magnet so that each line forms a closed curve.

The next step, and one of the greatest importance, is to arrange that these lines of force should indicate not only the direction, but also the magnitude of the magnetic force. This is done by establishing the convention that the number of lines of force passing through unit area shall be made to indicate the strength of the magnetic field at that point. Thus where the magnetic force is strong many lines will pass through unit area, that is, the lines will be crowded close together; on passing into a weaker region of the field there will be fewer lines in unit area, that is, the lines of force will spread out and get farther apart.

It must be clearly understood that it is by convention that lines of magnetic force are drawn so that the number passing through unit area indicates the field strength at that point, but that it is an experimental fact that all lines of force are closed curves, and that their number being chosen at any one point proportional to the field strength, that proportionality will hold at all other points along the paths of these lines.

§ 3. The Electric and Magnetic Circuits of the Dynamos.—A dynamo-electric machine will usually consist of (1) a part adapted to rotate, and carrying conductors so arranged that their rotation in a magnetic field will induce an E.M.F. in them, and of (2) a stationary part which will produce the necessary magnetic field. Alternatively the part producing the magnetic field may be made to revolve and the part carrying conductors in which an E.M.F. is generated may be stationary.

In either case there must be in every dynamo two different circuits to be considered: The magnetic circuit in which lines of magnetic force are induced, and the electric circuit which carries the current.

§ 4. **The Electric Circuit.**—Taking the electric circuit first, this consists of copper wire or copper bars insulated from one another and from all other metal by means of various insulating materials. To all these circuits Ohm's Law is applicable, *i.e.*, the current flowing through the circuit is equal to the E.M.F. divided by the resistance.

If C is the current, E the E.M.F., and ω the resistance, this is expressed by $C = \frac{E}{\omega}$. The resistance is independent of the current flowing through the circuit, but depends on the material of which it is composed, on its dimensions, and on its physical state. The resistance of most conductors for instance increases with an increase of temperature.

The resistance of any conductor varies inversely as its length and directly as its cross section. Thus, if the resistance of 100 feet of No. 16 S.W.G. copper wire, the section of which is ·00322 square inch, be ·254 ohm, the resistance of 200 feet will be $·254 \times 2 = ·508$ ohm, and the resistance of 100 feet of No. 14 S.W.G. copper wire, the section of which is ·00503 square inch, will be $\frac{·00322}{·00503} \times ·254 = ·162$ ohm.

Two or more conductors are said to be connected in series when they are so arranged that the current must flow through each of them in succession. They are said to be connected in parallel when the current divides between them so that a portion only of the

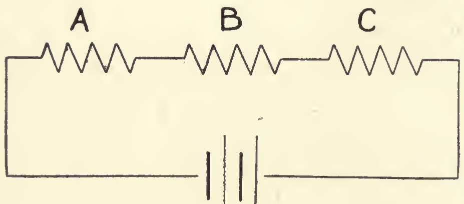


Fig. 3.

current flows through each of the conductors. Thus in Fig. 3 the resistances A, B, C are shown connected all in series; in Fig. 4 the same resistances are shown connected in parallel.

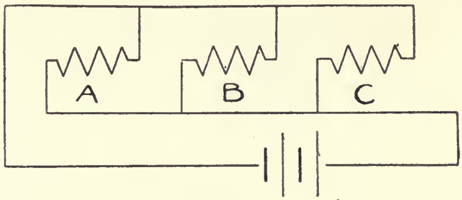


FIG. 4.

The resistance of a circuit made up of several conductors in series is obtained by adding together the resistances of the several conductors. In the case of conductors connected in parallel, it is the conductance, the reciprocal of the resistance, which must be added together to give the conductance of the complete circuit.

If a conductor having a resistance ω ohms be coupled in parallel with a conductor of resistance ω_1 ohms, the conductance of the joint circuit will be $\frac{1}{\omega} + \frac{1}{\omega_1} = \frac{\omega + \omega_1}{\omega\,\omega_1}$ and the resistance is the reciprocal of this, namely $\frac{\omega\,\omega_1}{\omega + \omega_1}$.

A special application of this formula, which will be frequently required in the following pages, occurs in the case of armature windings.

The total length of wire used in an armature winding is usually calculated and its resistance found; but the brushes carrying the current from the armature are very generally disposed in such a way that two, four, six, or more paths are available for the current to pass from brush to brush. In such cases the different possible paths are of equal lengths, and they are of course wound with material of the same section, and are thus of equal resistance. If there be two such paths, the resistance of each will be half that of the total winding, and the effect of connecting them in parallel will be equivalent to doubling the section, that is, again halving the resistance. The resistance of the armature from brush to brush will be $\frac{1}{4}$ of the total resistance of the winding. Similarly for four paths, the resistance will be $\frac{1}{16}$, and for six paths $\frac{1}{36}$ of the resistance of the whole length of wire, which makes up the electrical circuits.

The windings of the armature and of the magnets consist invariably of copper wires or bars. The use of this material is dictated by the fact that it has a lower specific resistance than any other material which can be commercially produced at a reasonable cost. This property enables the requisite number of turns of a suitable section to be got into smaller space than would be the case with a material of higher specific resistance. The resistance of electrical circuits on the dynamo must be kept as low as possible, because the watts lost in heating $C^2 \omega$ increase directly as the resistance.

A low resistance can only be obtained with a material of higher specific resistance by using an increased section, and an increased section of the winding means increased dimensions throughout the machine. For instance, iron has about seven times the specific resistance of copper; the windings of a dynamo might be carried out with iron wire instead of copper, but in order to keep the resistance at the same value, seven times the section of material would have to be used. The cost of the iron wire even at this increased section would be less than that of the copper actually used, but the dimensions of the machine would have to be so largely increased to find room for the increased bulk of windings that the cost, as a whole, would become prohibitive.

§ 5. **Electrical Units.**—The units used in electrical engineering for measuring E.M.F., current and resistance, are the Volt, the Ampere and the Ohm. The unit of E.M.F. is that produced in a conductor of unit length moving parallel to itself and with unit velocity in a magnetic field of unit intensity.

On the C.G.S. system this would be the E.M.F. produced in a conductor one centimeter long, moving with a velocity of one centimeter per second in a magnetic field, having one line of magnetic force per square centimeter. This E.M.F. would be very small, and the volt is, therefore, taken equal to 100,000,000 such units. The ampere is $\frac{1}{10}$ of the C.G.S. unit of current, and the ohm is equal to 1,000,000,000 C.G.S. units of resistance.

Thus the Volt $= 10^8$ C.G.S. units,
Ampere $= 10^{-1}$ C.G.S. units,
Ohm $= 10^9$ C.G.S. units,

and an E.M.F. of one volt acting through a resistance of one ohm gives a current of one ampere.

The rate at which energy is produced or dissipated in the electric circuit is measured in watts. The watt is the product of one volt by one ampere. Thus a generator giving a current of 100 amperes at a pressure of 100 volts is giving out energy to the circuit at the rate of $100 \times 100 = 10{,}000$ watts. The output of the generator is said to be 10,000 watts or ten kilowatts. Similarly, the electrical energy put into a resistance is all dissipated as heat; if the voltage at the ends of a resistance be 10 volts, and the current flowing through it 20 amperes, the rate of such dissipation will be $10 \times 20 = 200$ watts; in general

$$E \times C = \text{watts} \quad (1)$$

but by Ohm's Law

$$C = \frac{E}{\omega} \quad \therefore \quad E = C\omega$$

substituting in (1)

$$C^2 \omega = \text{watts.}$$

The rate at which energy is produced by a prime mover is usually measured in horse-power. The rate at which energy is put into a generator will, therefore, be measured in horse-power, but the rate at which electrical energy flows from its terminals is measured in watts ; it is frequently convenient to convert one of these units into the other, and this can be readily done by remembering that one horse-power is equal to 746 watts.

§ 6. The Magnetic Circuit.—Passing now to the magnetic circuit, the magnetic field required in dynamos is usually obtained by means of electro-magnets, that is, masses of iron or cast steel carrying a winding of copper wire through which a current circulates, the effect of the current being to cause lines of magnetic induction to pass round the magnetic circuit.

The number of turns of wire in a magnet winding multiplied by the current flowing through the winding is called the ampere-turns, and it is found that the number of lines of magnetic force which are linked with any coil is a function of the ampere-turns. The number of lines depends also on the material of which the magnetic circuit is composed. In air and most other materials the magnetic induction, that is, the number of lines per square centimeter, is directly proportional to the number of ampere-turns, but in iron or steel the induction due to a given number of ampere-turns is very largely increased. It is, therefore, advisable to make the magnetic circuit consist, to as large an extent as possible, of iron.

For example, in static transformers, which are used in alternating current work to change from one voltage to another, there are no moving parts, the magnetic circuit can be made entirely of iron, and in practice this course is invariably adopted. In dynamos and motors, however, one member must be capable of rotation, and as there must, therefore, be mechanical clearance between the rotating and the stationary parts, it is always necessary to have an air gap, *i.e.*, a part of the magnetic circuit where the lines of force have to pass from iron to iron, through air. Various considerations, which will be noticed in each separate case, settle what must be the length of magnetic path in air, but from the fact above noticed, the endeavour is always to keep this length as short as possible.

There is some similarity between the laws of the magnetic circuit and those of the electric circuit, in that the magnetic reluctance of any part of the magnetic circuit varies directly as the length, and inversely as the cross-section, of the magnetic path, and depends also on the material used, in the same way that the electric resistance varies directly as the length, and inversely as the cross-section, of the conductor, and varies with the material of which it is composed. The analogy does not, however, hold any farther, for the resistance is always independent of the current; the magnetic reluctance is independent of the magnetic induction only in air and other non-magnetic materials; in iron and steel the reluctance varies within wide limits, according to the magnetic density at which the material is worked.

It is, however, usual to look upon the magnetic circuit from a slightly different point of view. If H represents the magnetic force at any point, and B the magnetic induction at that point, that is the number of lines of magnetic force per square centimeter, the ratio $\frac{B}{H}$ is called the permeability of that part of the circuit. This ratio is usually denoted by the Greek letter μ.

In air and other non-magnetic materials, μ is equal to unity; that is, B, the magnetic induction, and H, the magnetic force, have the same numerical value. In iron and other magnetic materials, the value of μ varies with the value of B.

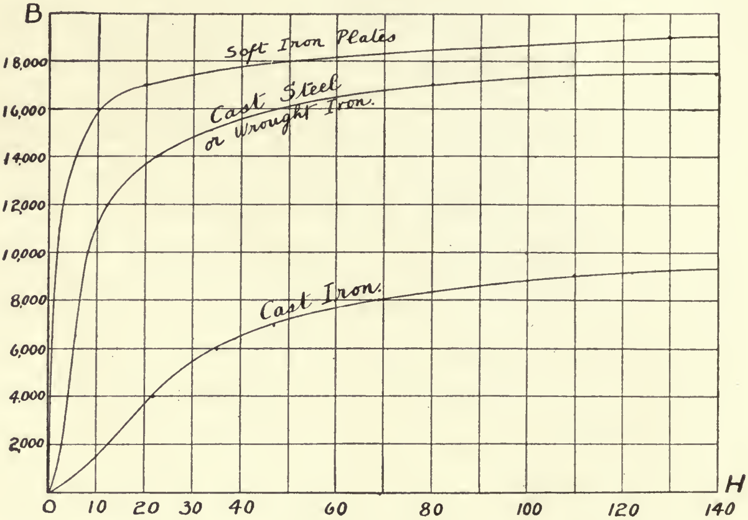


FIG. 5.

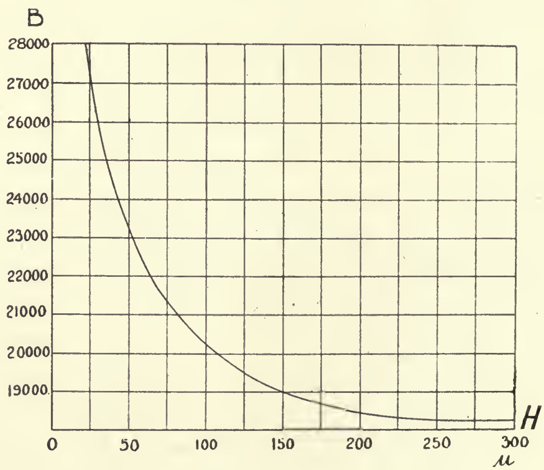


FIG. 6.

The curves in Fig. 5 give the relation between B and H for different kinds of material, and Fig. 6 gives values of the permeability μ for higher values of B where the scales become such as to make the plotting of the B H curve inconvenient. By the use of these curves it is easy to find the corresponding value of H for any given value of magnetic induction B.

In dynamo design, the problem of the magnetic circuit usually occurs as follows: The total number of lines of magnetic force required is known, and also the area and length of the different parts of the circuit, and it is required to find a suitable winding of the magnet coils to give the total flux required.

If A be the area of any part of the magnetic circuit, in square centimeters, and L its length in centimeters, and if N be the total number of lines of force required, then $\frac{N}{A} = B$ the number of lines per square centimeter. In air, H, the magnetic force = B, in iron or steel, the value of H corresponding to any value of B is found from the curves (see Fig. 5). Then $L \times H$ is the magneto-motive force required for this part of the circuit, and the sum of the terms L H found for every part of the circuit will be the total magneto-motive force to be provided.

Again, the number of turns of wire wound on any part of the circuit, multiplied by the current measured in amperes, flowing in the winding, is called the ampere turns in the winding, and this is proportional to the magneto-motive force $HL = \frac{4 \pi AT}{10}$ where A T represents ampere-turns or $AT = \frac{10}{4 \pi} HL$, that is, the ampere-turns required to give a magneto-motive force H L are equal to $\frac{10}{4} HL$. The value of $\frac{10}{4 \pi}$ is very nearly ·8, and it is sufficient for all practical purposes to remember that for any part of the magnetic circuit in which the magnetic force has the value H and the length of which is L, the ampere-turns required will be ·8 H L, and that the total ampere-turns required will be obtained by treating separately each part of the circuit having different values of H, and adding together the number of ampere-turns required for each.

§ 7. **Iron Losses—Hysteresis and Eddy Currents.**—In those iron parts of any machine where the magnetic induction varies at different times, there will be present losses due to eddy currents and to hysteresis. The former are due to the fact that the metal cutting magnetic lines has induced in it an E.M.F. which gives rise to currents flowing in the mass of the metal itself. These currents flowing against the resistance ω give rise to a loss of watts equal to $C^2 \omega$. This loss may easily become a serious one.

If, for instance, the armature core of a dynamo or motor were made of solid iron, the losses due to eddies would be extremely large, in fact might easily amount to many times the whole output of the machine. This effect is minimised by laminating the iron so as to increase the resistance and thus diminish the currents. For example, the armature core of a dynamo is made, not of a solid block, but of thin discs punched to the proper shape, and threaded on the shaft or on the spider. These discs, being more or less thoroughly insulated from one another, oppose large resistance to the currents which would tend to flow in directions parallel to the shaft. It can be shown that the watts lost in eddy currents vary directly as the square of the frequency, *i.e.*, as the square of the number of reversals of the magnetic force per second.

When iron is magnetised, the value of B corresponding to any given value of H varies not only with the quality of the material, but depends also on the previous history of the sample under test. The curves given in Fig. 5 give the relation between H and B, on the assumption that the specimen is thoroughly demagnetised between each reading. If this is not done, a different curve will be obtained. If the specimen is carried through a regular cycle of magnetisation by being subjected to a magnetic force, which is brought up to a considerable value in one direction, then goes back to zero and then increases to the same value in the opposite direction, it will be found that after this

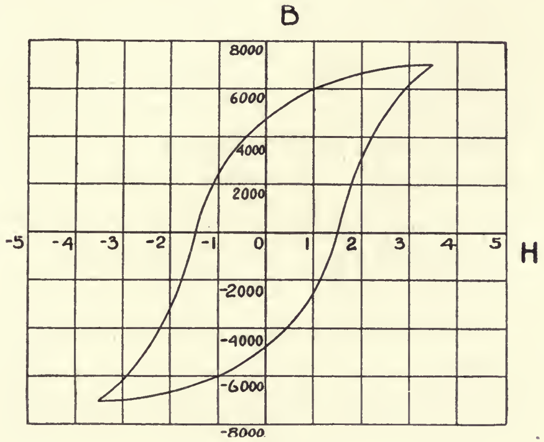


FIG. 7.

process has been repeated a few times, the magnetisation curve obtained will be similar to that shown in Fig. 7; that is, it will consist not of one curve, but of two curves forming a loop and enclosing a certain area. The value of B corresponding to a given value of H will be smaller when the iron is being magnetised than it is when it is being demagnetised. The area included between the two branches of this magnetisation curve represents work wasted in the iron in carrying it through the cycle of magnetisation. This work is converted into heat, and must be taken into account in the design. The amount of this loss evidently depends on the higher value of B to which the magnetisation of the iron is carried. It depends also, to a very large extent, on the magnetic quality of the iron used.

It is frequently assumed that the hysteresis for any given sample is proportional to $B^{1.6}$, and this is probably sufficiently accurate within the range used in ordinary practice, although the accuracy of this formula for wide ranges has been largely disputed. The hysteresis loss being constant for one cycle will naturally vary as the total number of the reversals per second.

It should be noted that whilst the $C^2 \omega$ losses in the electric circuit are constantly present so long as the current C is flowing through the circuit, the iron losses in the magnetic circuit, whether eddy current losses or hysteresis, occur only at such times as the magnetic field is varying either in intensity or in direction.

Thus in the field magnets of a dynamo excited with continuous current, when once the excitation has reached a constant value, the magnetic field also remains constant, and there is no loss from either of the above causes; in the iron body of a rotating armature, on the other hand, each element of the core is being constantly carried from one magnetic field to others of different intensities and directions, and therefore both eddy losses and hysteresis have to be reckoned with.

§ 8. Value of E.M.F. Generated.—The unit of E.M.F. is defined (see p. 9) as that produced in a conductor of unit length, moving in a direction at right angles to its length and to the lines of force and with unit velocity in a magnetic field of unit density, *i.e.*, in a field having one line of magnetic force passing through each unit area. The E.M.F. increases with the length of conductor, the velocity, and the density of the magnetic field. Therefore, a conductor of length l moving with velocity v in a field having

B lines of force per unit area will have generated in it $v \times l \times$ B units of E.M.F.

Again, since v is the velocity, the distance passed over by any point of the conductor in unit time is v, and vl represents the area swept over by the conductor in this time, and since there are B lines per unit area, the total number of lines of magnetic force cut by the conductor in unit time is $v \times l \times$ B, but this is the expression already found for the value of the E.M.F., and therefore the E.M.F. is equal to the number of lines of magnetic force cut by the conductor in unit time. If, as is usual, the volt be used as the unit of E.M.F., and the second as the unit of time, the E.M.F. generated in a conductor moving through a magnetic field will be equal to the total number of lines cut per second, divided by 100,000,000 (10^8), since this is the number of C.G.S. units of E.M.F. in one volt.

§ 9. Electric and Magnetic Properties of Materials.—Numerical values of the resistance of the copper used can be calculated from the resistance of one cubic inch of copper = ·000000667 ohm, but they are more conveniently taken from a table of the properties of copper wire, such as are published in many textbooks, and by most manufacturers of cable and wire. A specimen table is given in the Appendix. The resistance of copper increases by about ·38% per degree Centigrade, or ·21% per degree Fahrenheit, and it is usual to calculate the resistance of the copper windings of a dynamo at the highest temperature which it is expected the machine will reach. Another column is therefore added to the table in which 20% is added to the resistance, this allowance being sufficient to cover the increase of resistance, due to a temperature of about 130° F., and also the increase due to the stretching of the wire, which almost invariably takes place whilst it is being wound on the machine.

The choice of material from which the magnetic circuit shall be constructed is very limited. The fact that the permeability of iron is much greater than that of any other known material compels the use of iron in some form or another in all parts of the magnetic circuit. Magnets are therefore built of wrought iron, cast steel, or cast iron. The magnetic properties of the iron used vary considerably in different specimens, the curves given in Fig. 5, however, are the mean of those usually obtained in practice and will be used throughout the calculations.

§ **10. Insulating Materials.**—It is not only necessary to provide in the copper windings a free path for the electric current; it is also necessary that means should be taken to prevent the current from straying through paths where it is not required. For this purpose, insulating materials of various kinds are used. There are many materials which offer a large resistance to the passage of electricity; wood, paper, fabrics of silk or cotton, ebonite, glass, marble, slate, mica, and asbestos, for example, are all used in different classes of electrical work as insulators.

Insulation may fail by allowing the electric current to pass through its substance, or to creep over its surface; the presence of moisture (water is a comparatively good conductor of electricity) or of dirt will spoil the insulating properties of most of the materials mentioned above by allowing the current to creep through them or over their surfaces.

The resistance which a material opposes to the current passing through it or over its surface can be measured in ohms by any of the well-known methods for measuring high resistances. For instance, the resistance measured between the conducting circuit of a dynamo and some metal part of the frame, not intended to carry current, indicates the amount of current which would leak through to the frame under working conditions. This resistance is called the insulation resistance of the machine, and is usually measured in megohms, a megohm being equal to one million ohms.

In addition to this it is found that any given thickness of any particular insulating material will stand only a certain E.M.F. applied to its opposite surfaces; if this be exceeded the insulation breaks down, the material is mechanically punctured, a spark passes, and an arc may be maintained.

However high the insulation resistance of any apparatus may be, it is no guarantee that it will not break down in this way. In addition to a high insulation resistance, the material used must also have sufficient dielectric strength to resist the E.M.F. at which the machine is to be used. A material has a good dielectric strength, a moderate thickness of which requires considerable E.M.F. to break it down.

The chief requirements of a good insulating materia then, are, that it should give a high insulation resistance and have good dielectric strength; it should not be hygroscopic, that is, it should not readily absorb moisture. Mechanical strength is required in many parts of a dynamo, and in some places the material must also be flexible, so that it can be bent to the required shape without breaking or cracking, and it must stand a fairly high temperature without deteriorating.

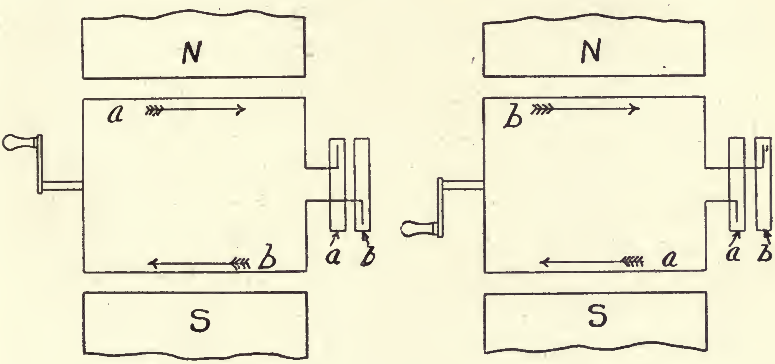
There is no single material which even approximately fulfils all these conditions. Cotton, either wrapped directly on the wire or used in the form of tape, is useful because of its flexibility, but it has very little dielectric strength, and readily absorbs moisture. Silk has much the same properties as cotton; it is a better insulator and is sometimes used in dynamos where space is limited, as a thinner covering of silk can be put on the wire than is practicable with cotton. The very much greater cost of silk, however, prohibits its general use. Paper, cardboard, and various materials on the market, which are made from wood pulp or similar preparations, are tough and stand fairly rough handling in the building of a machine; most of them, however, absorb moisture. Mica is excellent in its dielectric strength and is not very hygroscopic. It is, however, unsuitable for many purposes, because of its want of flexibility. India-rubber and vulcanite soften at the temperatures to which a dynamo rises, and preparations such as fibre and many other manufactured insulations warp when exposed to these temperatures.

The usual method of insulating the circuits in the dynamo is to use a cotton covering on the conductor, either braided or simply double cotton-covered. This is by itself sufficient to insulate from one another the turns of the coil. Corners and bends are, however, protected by a further wrapping of cotton tape, and to counteract the moisture-absorbing properties of the cotton the coils are frequently dipped in varnish and baked before being put on to the machine. The metal parts of the machines, whether armature core or spools for the field coils, are further insulated before the coils are put on. For pressures up to 500 volts, paper or similar material such as presspahn, leatheroid, etc., is found to be sufficient, but for machines to be worked at high voltages, or for such as will be subject to be worked in very high temperatures, or to be exposed to damp, mica is the best material for this purpose.

CHAPTER II

VARIOUS TYPES OF DYNAMOS AND MOTORS

§ 1. **Elementary Armature.**—A loop of wire bent into a rectangular shape, as shown in Fig. 8, and mounted so as to be capable of rotation between the two poles of a magnet is the simplest form from which the dynamo may be considered to have been evolved. As the loop is re-



A B

FIG. 8.

volved, the two sides marked *a* and *b* will cut lines of magnetic force, and will have an E.M.F. induced in them; if the loop is closed on itself, a current will circulate through it, but it will be unavailable for any useful work as it never leaves the closed loop. In order that the current should be made use of, it must be carried to some external circuit in which it is to be employed, and since the loop is revolving it is obvious that the current can only be carried to a fixed external circuit by means of sliding contacts.

In the figure the loop is shown not closed in itself, but having its ends connected to two rings *a* and *b*; on each of these rings rests a copper or carbon brush which collects the current from the ring, and carries it to the outer circuit.

A little consideration will show that the current produced by such an apparatus will be constantly varying in direction. If the direction of rotation be such that the top of the loop *a* is moving from back to front of the paper, *i.e.*, towards the observer, the E.M.F. induced in it will be from left to right, in the bottom bar it will be from right to left. Thus the current will enter the external circuit at the ring *a* and leave it at *b*; *i.e.*, the brush at *a* will be the positive (+) terminal, that at *b* the negative (−) terminal. But when half a revolution has been accomplished, the loop will have got into the position shown at A, Fig. 8, and the ring *b* will now be positive, the ring *a* negative, and the direction of the current in the external circuit will be reversed. This reversal will take place at every half revolution of the loop. The current generated in this way is known as an alternating current, and is constantly varying in magnitude and direction.

§ 2. A Commutator Necessary to give Continuous Current.—In order to produce a continuous current, which flows always in the same direction, it is necessary to provide, in connection with the armature winding, a commutator, a device to alter the connections between the revolving loop and the external circuit in such a way as to rectify the current in the latter, and make it flow always in the same direction.

The simplest way to do this is to substitute for the two rings *a* and *b*, two half rings, the brushes being arranged so that they make contact alternately with the two half rings, and pass from the one to the other as the loop comes into the horizontal plane (see Fig. 9).

By this arrangement, although the half rings *a* and *b* will be alternately positive and negative, just as were the rings *a* and *b* in Fig. 8, the brush *c* will always be in contact with that half ring which is positive, and brush *d* with that half ring which is negative; the direction of the current in the outer circuit will always be from *c* to *d*.

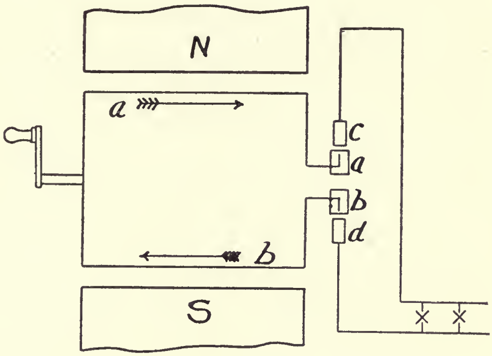


FIG. 9.

In order to pass from this elementary dynamo, consisting of a single loop of wire revolving in a uniform magnetic field, to the present-day machines, two chief modifications are required. Firstly the space within the loop *a b* must be filled with iron, so as to decrease the magnetic reluctance, and allow of the use of a larger number of magnetic lines, and secondly the number of conductors must be largely increased. Under ordinary conditions the E.M.F. generated in a single loop is insufficient for practical purposes.

The number of conductors could be increased by simply winding several turns of wire instead of one only in the loop, but it will be seen later that this can only be done to a limited extent, as many turns between commutator parts lead to sparking at the brushes.

The expedient generally employed is to use not one loop, but several, the ends of each loop being brought out to two segments of a ring which together form a commutator of four, six, or more parts, according to the number of loops. Thus the commutator is evolved, consisting of many segments insulated from one another and built up to form a ring or cylinder on which the brushes rest.

§ **3. Open- and Closed-Coil Armatures.**—The armature coils consisting of one or more turns of wire are connected to these segments, and this can be done in two ways; if the ends of each coil are brought out to two commutator segments which are not connected to any other coil, the armature is known as an open-coil armature (see Fig. 10). Examples of this method are the Brush and the Thomson-Houston arc lighters. The armature coils brought to the commutator are in circuit with the brushes whilst they are giving current in the right direction, and are cut out of the circuit when the reversal takes place. This method was found extremely unsatisfactory from the sparking point of view, the coil which was being cut out had its circuit broken at the brush whilst carrying full current, and violent sparking ensued.

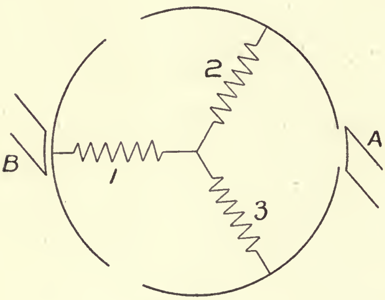


FIG. 10.

All present-day machines are invariably constructed on the closed-coil principle; that is to say, the ends of the coils are brought to a commutator blade in such a way that each coil is short-circuited by the brush whilst the current in it is undergoing reversal. The number of commutator parts is made large so that there should be as few turns of wire as possible between two adjacent commutator parts.

The closed-coil winding is obtained by connecting one coil between each two adjacent commutator segments, as shown in Fig. 11, so that there is a complete circuit through the armature winding independently of the brushes.

The difference in the process of reversal is easily seen. Say the current flows into the armature at the brush A and out at B. In Fig. 10 the current will divide between the coils 2 and 3, then the whole current will pass through coil 1, and out at brush B. When the armature has moved through a small angle the brush A will break contact with commutator part 2, the whole of the current flowing through coil 2 (half the current taken by the machine) will be violently broken, and sparking at the brush will take place.

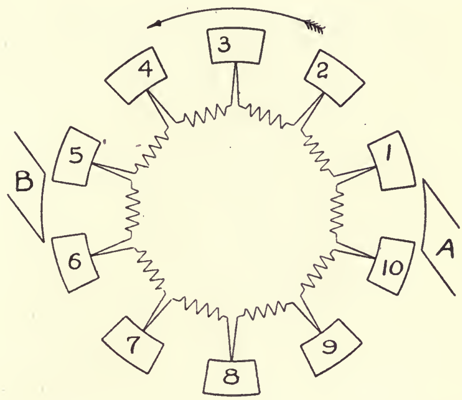


FIG. 11.

In Fig. 11, on the other hand, the current again divides into two parts, half flowing through 1, 2, 3, 4 and 5, to brush B, and the other through 10, 9, 8, 7, 6, but the circuit through any coil is at no time broken. As the armature moves through a small angle, brush A leaves contact with part 1, but it is still in contact with part 10, and the current which flowed from the brush to part 1 can now pass to part 10 and from 10 through one armature coil to part 1 of the commutator; there is therefore no violent interruption.

It must, however, be noticed that each coil as it passes under the brush is momentarily short-circuited by the brush, and that during this period of short circuit the direction of the current in the coil is reversed. For instance, in the position shown in the figure the current flows from part 10 to part 9, through the coil connecting these parts, but when the rotation of the armature has carried this coil into the upper half of the figure, the direction of the current being still from brush A to brush B, the direction in the coil under observation will now be from part 9 to part 10; the current will have changed its direction as the coil passed under the brush.

If the machine is properly proportioned, this reversal will be effected smoothly, but if the proper proportions are not observed, the reversal of the current will not have been completed by the time the commutator part leaves the brush and sparking at the brushes will ensue.

§ 4. **Drum and Ring Armatures.**—The armature winding of a C.C. machine consists of many bars or wires laid side by side on the armature core, and these must all be connected together in series so that the E.M.F. generated in each will add together to give the total required. In Fig. 12, *a*, *b*, *c*, *d* represent diagrammatically four neighbouring bars of the winding; since these are all simultaneously in the same region of the magnetic field the E.M.Fs. generated in them will be in the same direction. Let the arrows indicate this direction; in order that the bars should be combined into a winding so that their E.M.Fs. shall add together, it is evidently necessary that the top end of *a* should be joined to the bottom of *b*, and so on, as indicated by the dotted lines. But this connection cannot be made on the face of the armature; if it were, an E.M.F. would be induced in the

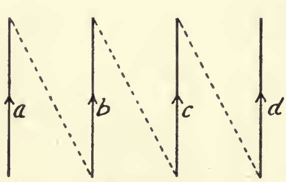


FIG. 12.

connecting wire opposing that in the bars, and the total result would be no E.M.F. at the ends of the winding.

There are two ways in which the connection can be made. The winding may be passed through the interior of the core, where there is no magnetic field; the armature will then resemble an iron ring with insulated copper wire wound spirally round and round it. This type of armature is known as a ring armature. The principle of this winding is shown in Fig. 13. Or the connections may be laid on the surface of the core, provided care be taken that the return wire shall pass under a pole of opposite polarity to that under which the bar is situated, that is, the pitch of the winding must be approximately equal to the pole pitch. A winding disposed in this way on the surface of the armature gives what is known as a drum armature, a smooth drum if it is laid on the surface, a slotted drum if the winding is put in grooves or slots cut in the iron.

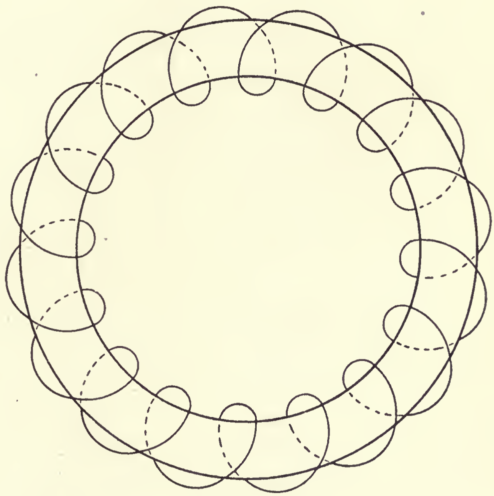


FIG. 13.

In continuous-current machines it is necessary that perfect symmetry should be preserved in the winding, and schemes of connections are always arranged with this end in view. For machines in which the current is not very great, the winding usually consists of several turns of copper wire insulated with a cotton covering. The wire is generally wound into coils having the necessary number of turns, and made into a suitable shape, and the finished coils are then put into place on the armature. Fig. 14 shows one of the shapes which such a coil may take ; the portion A, which is shown full, lies at the top of a slot, whilst the portion B, shown dotted, will lie at the bottom of another slot. All the coils on the armature are exactly similar, and it will be seen that this forms an absolutely symmetrical winding. The two ends of the coil are connected to the commutator.

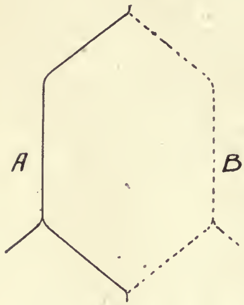


FIG. 14.

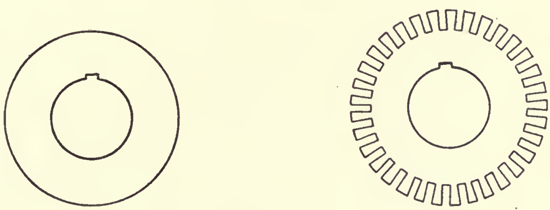
Ring armatures were formerly frequently used. They have the advantage of being easily repaired, each coil occupying its own portion of the core, and being quite independent of the others. Any damaged coils can therefore be easily removed and replaced. In the case of the drum armature the end connections of one coil cross below the connections of many other coils, and it is impossible to remove a damaged one without lifting a considerable number of sound coils.

On the other hand, the ring armatures where the wire passes through the interior of the core must of necessity be wound by hand, the wire being at each turn threaded through the hole in the centre of the core. In the case of the drum winding, coils can be wound and formed beforehand and then be assembled in place. This is a great advantage, not only in saving time, but in insuring uniform insulation. It is claimed for the drum winding that the length of idle wire—that is, wire which is not in the magnetic field—is less than on a ring armature, and this no doubt is generally the case, although in certain sizes of machines, by giving the core a proper section, the amount of idle wire in a ring armature is not much greater than on the corresponding drum.

§ **5. Smooth-Core and Slotted Armatures.**—In a smooth-core armature the conductors are laid on the surface of the iron core ; in a slotted armature the discs are punched out, so that when they are assembled, grooves or slots are formed from end to end of the armature core into which the bars or wires are laid.

In Fig. 15 discs are shown for a smooth-core armature (A) and for a slotted armature (B).

In a smooth-core armature the diameter of the iron core must be kept small enough to ensure the necessary



A FIG. 15. B

mechanical clearance, and also to allow room for the copper and the necessary insulation.

In the slotted armature the winding lies below the cylindrical surface of the armature, and the iron of the armature can, therefore, be brought close to that of the magnets, mechanical clearance only being required. The part of the magnetic circuit lying in air is therefore shortened, and fewer ampere-turns are required on the magnets. The advantage of the slotted armature in this respect is increased, from the fact that the surface of the winding in the case of a smooth-core armature is always more or less uneven ; the wires are laid on separately, and do not form as true a cylindrical surface as is obtained with a slotted armature; the mechanical clearance to be allowed is, therefore, greater with smooth-core than with slotted armatures.

Mechanically also, the construction employing a slotted armature is much more satisfactory. On a smooth-core machine, the wires are held in place only by friction, or by driving horns, that is, projections, usually of wood, which are let into the core at intervals in order to give a direct drive to the wires. It is evidently much better to have the wires positively driven by being placed in slots, and, as a matter of fact, the advantage is even greater than would at first sight appear, for it can be shown, and it has been proved experimentally, that wires buried in the iron of the core are relieved of practically all the magnetic pull to which they would otherwise be subject. This is easily realised when it is considered that there is only a very small magnetic field in the slots, and that practically the whole of the lines of magnetic force are carried by the teeth; thus the magnetic circumferential drag, which in a smooth-core armature is taken by the wires, is in a slotted armature transferred to the teeth.

As a set-off to these advantages it must be noted that the self-induction of a winding carried in the slots is much greater than that of a winding which is in the greater part surrounded by air, and thus, other things being equal, the reversal of the current will be more difficult in a slotted than in a smooth-core armature, and there will accordingly be a greater tendency to spark. Thus, while the old smooth-core machines were generally used with copper brushes, slotted armatures seldom work well with copper, but have to be fitted with carbon brushes with which the commutation is much easier. The influence of self-induction and also of carbon brushes on commutation will be seen in future chapters.

§ 6. **Bipolar and Multipolar Machines.**—The magnet system in which an armature is to run may be arranged with one or more pairs of poles.

In alternating-current work, the number of poles is rigidly fixed by the relation between the speed of rotation and the periodicity of the current; in continuous-current work there is no such rigidity and there is, therefore, much more latitude in the choice of the number of poles.

Dynamos having two poles only are said to be of the Bipolar type, and those having more than two poles, of the Multipolar type.

For many years continuous-current generators and motors were most frequently made of the bipolar type. Multipolar machines are now generally used, and a two-pole construction is uncommon. The necessity for increasing the number of poles became apparent as soon as slotted armatures were introduced, because the decreased number of ampere-turns required on the magnets gave an undesirable preponderance to the armature ampere-turns. As will be seen, strong armature reaction is undesirable, and by increasing the number of poles the armature ampere-turns per pole—that is, the armature-turns opposing the ampere-turns—on one magnet coil can be diminished, hence the general use of multipolar machines. It is, nevertheless, true that in small machines where the armature ampere-turns are in any case not excessive, the two-pole construction could in many cases be adopted. It is found, however, that a saving in material can generally be effected by using four instead of two poles, and most machines of recent construction of any size larger than a toy have at least four poles.

In the early days of dynamo-electric machinery many different types of armatures and field magnets were in use. At the present time multipolar magnets with slotted drum-wound armature are almost universal, and it is to machines of this type that the following descriptions and calculations will chiefly refer.

The general appearance of a continuous-current dynamo of the multipolar type with a slotted drum armature, such as would be constructed at present, is shown in Fig. 24 at the end of this chapter, and an outline sketch of the machine with the names applied to the different parts is given in Fig. 25; reference to this diagram will enable the descriptions in the ensuing chapters to be readily followed.

The winding of a slotted drum armature may be carried out with round wire, or with bars of a square or rectangular section. If the current to be carried is not large, and the section of the conductor is consequently small, round wires are preferred. A square or rectangular wire would, it is true, pack closer in the slot, and thus allow a larger section of copper to be got into the same space, but square wires of small section are very apt to get a twist, especially where any bending occurs, and instead of lying parallel to one another, will, on account of the twist, have a great tendency to cut through the insulation at the corner, and cut into one another; this disadvantage is altogether absent in the case of round wires, and they are consequently generally employed for windings of small current-carrying capacity.

When, however, the size of wire becomes so large as to make it difficult to bend, bars preferably of a rectangular section are used; with the increased section the number of turns per coil naturally become less, and it is therefore practicable to make each turn out of one bar and connect the separate turns by means of soldered joints.

In either case, whether made of wire or bars, all the coils must be of exactly the same shape in order to ensure that the winding shall be symmetrical, and, since it is also necessary that the two sides of a coil should be simultaneously under poles of opposite polarity (see section 4 of this chapter), the span of the coil must be equal to the pole pitch, and it will in general span over several slots; arrangement must therefore be made so that the end connections of the coils outside the slots pack together without interfering with the wires coming out of intermediate slots. This is done by shaping the coils as in Fig. 14; the bar shown full (as we have said) lies at the top of the slot, the part dotted at the bottom of another slot, whilst of the end connections the part shown full crosses over and lies in a plane above the dotted parts of the coils coming out of intermediate slots.

It is always necessary that the end connections should lie in at least two different planes, as here indicated, but it is not necessary that these two planes should be tangential to the armature surface. When they are so arranged, and this is the more usual practice at present, the winding is called a developed end winding and sometimes a barrel end winding. The two planes in which the end connections are disposed may, however, be sloped at any angle; in the limit, instead of being parallel to the armature shaft, they become perpendicular to it, and the coil then has the appearance shown in Fig. 16. Any angle between the positions of Fig. 14 and Fig. 16 is permissible, and such intermediate positions have frequently been used for end connections.

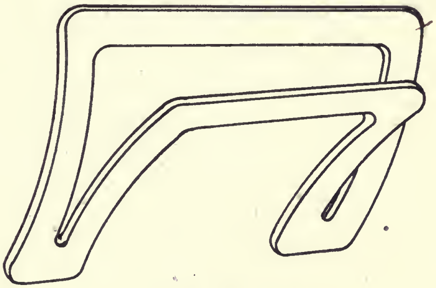


FIG. 16.

The use of developed end windings has a tendency somewhat to lengthen the armature, but the coils are much more easily formed and assembled in place, and this ease of handling is of sufficient advantage to have made this style of winding almost universal.

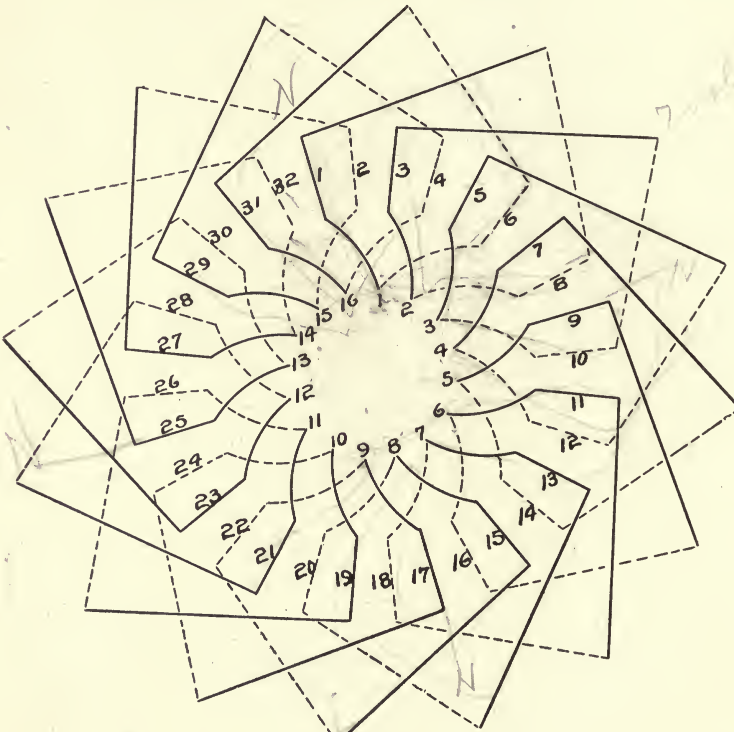


FIG. 17.

§ 7. Lap and Wave Winding.—On multipolar machines, that is machines which have more than two poles, two different systems of connecting the different coils together may be employed, known respectively as wave winding and lap winding. In either case the distance between A and B (see Fig. 14) must be equal to the pole pitch, that is, in a four-pole machine the distance A B will be $\frac{1}{4}$

of the circumference; in a six-pole it will be $\frac{1}{6}$, and so on. But whereas in a lap-wound machine the two ends of A and B are connected to adjacent commutator parts, in the case of wave winding they are connected to commutator parts the distance between which is approximately equal to twice the pole pitch.

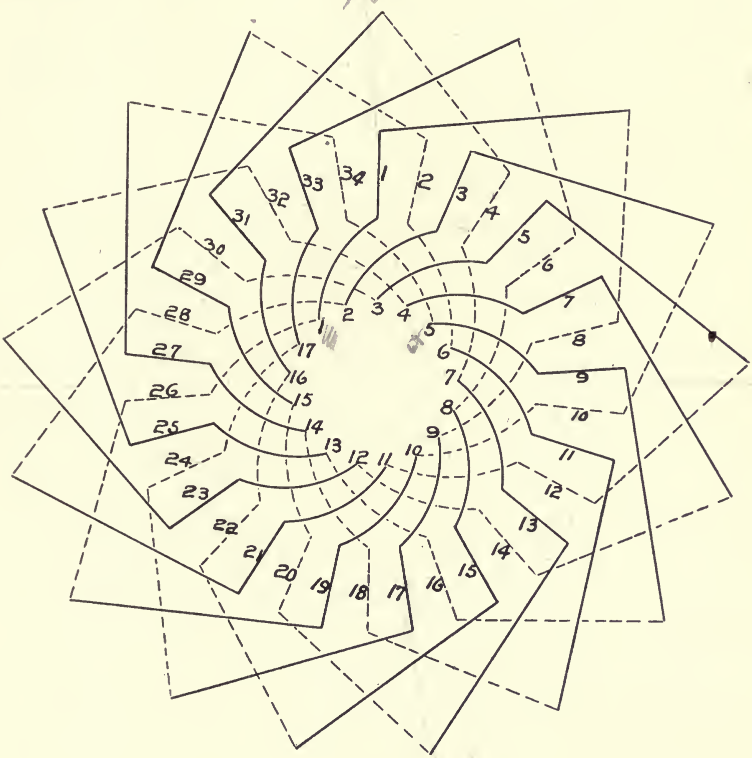


Fig. 18.

The properties of wave and lap winding can be more fully studied by the help of diagrams and what are known as winding tables. In order to get full advantage of such diagrams and tables the student should construct them for himself, since during the process of drawing the diagram or writing the table many points emerge very clearly which are to a certain extent obscured by the overlapping of coils on the finished diagram or table.

Figs. 17 and 18 show diagrammatically the connections of a four-pole armature, lap- and wave-wound respectively. The radial lines indicate the bars lying in the slots; the lines outside the circle of bars represent connections at the back end of the armature, and the curved lines inside the circle the connections at the commutator end; each of the numbers in the inner circle indicates a commutator part.

In the case of bar winding, having only one turn per commutator part, each bar must be shown on the diagram, but where there are two or more turns per commutator part, it is only necessary to indicate the ends of each coil which go to the commutator, and for winding purposes. Each coil, however many turns it may consist of, may be considered as equivalent to two bars.

There are some limitations to the number of bars which may be used on an armature. Firstly, in any winding, since two bars are required to make up a coil, and since two bars are connected to each commutator segment, it is evident that the number of bars must be even.

In a lap winding this is the only restriction, and any even number of bars may be used.

In a wave winding it is in addition necessary that the following relation should hold:—If y is the average pitch and P the number of poles, n the total number of bars, then

$$n = \mathrm{P}y \pm 2.$$

The bars round the armature being numbered consecutively, the pitch is defined as the number of bars intervening between the two bars to be connected together. Thus if bar 1 and bar 36 are to be connected together at the back end of the armature, the pitch at the back will be 35;

the front pitch may be the same as the back or it may differ from it; in either case, the term average pitch, as used above, is to be interpreted as the mean of the back and front pitch.

In a four-pole machine $P = 4$, let the average pitch be 36, then $Py = 144$, and 142 or 146 are possible numbers for a wave winding, but 144 is not. In the same way a six-pole machine may have an average pitch of 70, in this case $Py = 420$; then 418 and 422 are possible numbers for a wave winding, but 420 is not. Choosing slightly different values for the pitch, say 69 and 71, values of $Py = 414$ and 426 are obtained, and it is found that 412 or 416, 424 or 428 bars may be used. So that for a six-pole wave winding requiring about 420 bars, it is possible to use 412, 416, 418, 422, 424, or 428 bars, but 414, 420, and 426 are not practicable numbers.

§ 8. **Armature Winding Tables.**—The fact already noticed that the end connections must lie in two different planes is clearly brought out in Figs. 17 and 18, where the full lines show bars at the top of the slots and their connections, and the dotted lines show bars at the bottom of the slots and their connections. It is seen that the end connections cross one another at many points, but that in all cases it is a full line that crosses a dotted line; in no case do full lines cross one another or dotted lines cross one another; the significance of this being that while the two layers of connections cross one another all the bars in either of the layers lie parallel to one another.

At both ends of the armature it is necessary that a top bar should connect with a bottom bar. In numbering the bars, if a top bar be numbered 1, the bottom bar below it will be 2, the next top bar 3, and so on; it follows from this that all top bars will be indicated by odd numbers and all bottom bars by even numbers, and since a top bar must always connect with a bottom bar and *vice versâ*, the pitch both front and back must be an odd number. Fig. 19 shows three slots of an armature having six bars

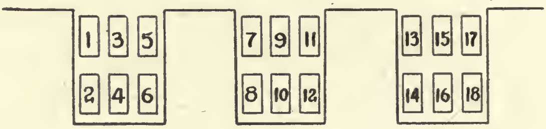


FIG. 19.

per slot, and indicates the order to be followed in numbering the bars.

Since the symmetry of the winding requires that all coils have the same shape, the back pitch must be the same at all points of the armature, so also must the front pitch; but the back and front pitch may differ from one another.

Fig. 17 shows a four-pole lap winding for an armature having 32 bars. This is a much smaller number than would actually be used in ordinary circumstances: the number of bars on a machine frequently amounts to several hundreds; but a diagram drawn with such a number would become so complicated as to be valueless in explaining the principles of armature connections.

Starting to trace the current from the commutator end of bar 1, the back connections lead to bar 8. It has already been pointed out that the bars connected together should be simultaneously under poles of opposite polarity, and this condition is fulfilled in the present case, by connecting together bar 1 and bar 8; the back pitch is thus 7.

In order to obtain a lap winding, the front pitch must differ from the back pitch by 2; and it must be counted in the opposite direction. In this case it must be either 5 or 9; counting 5 bars back from 8, bar 3 is the next to be picked up; using 9 as the pitch, it is bar 31 which is reached. Note that in either case, one of the top bars immediately adjacent to No. 1 is the next top bar to be included in the circuit. This is the necessary characteristic of a simple lap winding.

As it is advisable to keep the end connections as short as possible, it is usual to choose the smaller of the two possible numbers for the front pitch ; in this case 5, and the order of the bars as they are reached on following out the winding, can be set out in the following table, B indicating the back pitch, F the front pitch.

B = 7	F = 5.
+ 1	8
3	10
5	12
7	14
− 9	16
11	18
13	20
15	22
+ 17	24
19	26
21	28
23	30
− 25	32
27	2
29	4
31	6
—	—
1	8

The second column is obtained by adding 7, the back pitch, to the number in the first column, and the figures in the first column are obtained by subtracting 5, the front pitch, from the last figure in the second column.

Thus $1 + 7 = 8$, $8 - 5 = 3$, $3 + 7 = 10$, $10 - 5 = 5$, and so on.

It will be seen that after a certain number of lines, bar 1 is again reached, and if the process be continued the table will simply repeat. If the whole of the bars have been picked up before this happens, that is if every number from 1 to 32 appears in the table before 1 is again reached, the winding is correct ; if not, some mistake has been made, or the value of the front or back pitch has been wrongly chosen.

The diagram and the table should be compared, when it will be seen that the two correspond, and that the bars occur in the same order in each.

On a lap-wound four-pole armature, four brushes will be placed on the commutator, dividing it into four equal parts ; say the brushes simultaneously touch commutator parts 1, 5, 9, and 13, call the brushes on 1 and 9 positive brushes, then those on 5 and 13 will be negative ; the + and − signs in the table indicate the position of the brushes with respect to the winding. The current passing from the + to the − brushes has four possible paths through the winding ; it can flow from commutator part No. 1, through bars 1, 8, 3, 10, 5, 12, 7, 14, or through 6, 31, 4, 29, 2, 27, 32, 25, or the current may flow from commutator part No. 9, which is also in contact with a positive brush through bars 17, 24, 19, 26, 21, 28, 23, or through bars 22, 15, 20, 13, 18, 11, 16, 9. There are thus four circuits in parallel through the armature. In a simple lap winding, the number of paths through the armature is always equal to the number of poles.

Fig. 18 shows a wave winding, the same conventions hold as in Fig. 17 ; in the case of a wave winding, 32 is not a possible number of bars for a four-pole machine, $n = \text{P}y \pm 2$ (see page 36) here $\text{P} = 4$, assume $y = 8$, then P $y = 32$, and n, the number of bars, must be 30 or 34, the latter number has been chosen. For wave windings, the back and front pitch are counted in the same direction, they must both of course be odd, and the mean of the two must in this case be 8, thus 7 and 9 may be chosen, and a table written out on the same lines as the table for the lap winding.

B = 7	F = 9	B = 7	F = 9.
+ 1	8	+17	24
33	6	15	22
31	4	13	20
29	2	11	18
27	34	− 9	16
− 25	32	7	14
23	30	5	12
21	28	3	10
19	26	1	8

This table is obtained by adding the pitch, whether back or front, to the preceding number; there is no subtracting, since the pitch is in both cases counted in the same direction.

If the brushes are again considered to make contact on commutator parts 1, 5, 9, and 13, and the circuit traced out, it will be found that there are now only two paths for the current from positive to negative brushes; namely, the path 1, 26, 19, 10, 3, 28, 21, 12, 5, 30, 23, 14, 7, 32, 25, and the path 17, 24, 33, 6, 15, 22, 31, 20, 29, 2, 11, 18, 27, 34. The positive brushes on commutator parts 1 and 19 are connected together by bars 1 and 8, which are at the time out of the magnetic field, and therefore generating no E.M.F. One of these brushes may therefore be lifted off the commutator without in any way affecting the distribution of the current; the same applies to the negative brushes.

In all simple wave windings, there are only two paths in parallel through the armature, whatever the number of poles, and the current may be collected at the commutator, either at two points or at a number of points equal to the number of poles. In the case of wave winding, the two positive brushes or the two negative brushes, taking the case of a four-pole machine, are electrically at the same point of the armature winding; although they are diametrically opposite one another on the commutator, the segments with which they are in contact are connected together by one armature coil only, and the bars of which this coil consists are at the time in the middle of the space between adjacent poles and are giving no E.M.F. The current divides, therefore, between the two brushes in proportions which are settled only by the contact resistance of the brushes and the resistance of their connections to the terminals. There is no positive force causing the current to divide equally between the two brushes. This is found in practice to be of little importance in small machines, but in large machines it may become a very serious matter; as one brush may take a large part or even the whole of the current, it is consequently badly over-loaded, sparking takes place, and the brush may get extremely hot.

In a lap winding, this difficulty does not occur; to pass from one positive brush to the other, half the coils of the armature must be passed through, and each brush has to take its share of the current coming from the adjacent quarters of the winding. The whole of the E.M.F. generated in these coils compels the current to flow through the brush. There is, therefore, in this case, no chance of the brushes getting unequally loaded on account of their resistances being different; but one or other of the windings which are put in parallel by the brushes may have generated in it an E.M.F. slightly in excess of the other windings. This will cause it to take an excessive share of the current, and this excessive current will pass from the commutator into one set of brushes.

This can be remedied by the use of what are known as equalising rings. Equalising rings consist of connections joining all those bars which are under, say a positive brush, at the same instant. The number of such connections is not very material; every third or fourth bar may be joined to all other bars at the same potential. These connections keep the excess current from passing out to the brush and spoiling the commutator and, further, by giving a free path for the excess current generated, they increase the armature reaction to such an extent as greatly to modify, if not totally to do away with, the uneven distribution of magnetic field which originally caused too large an E.M.F. to be generated in that part of the winding. Thus the fault of too great a current in any one part of the winding automatically corrects itself.

By varying the pitch at the front or back, various other schemes of winding may be obtained, having the effect of multiplying by 2, 3 or more the number of paths in parallel. These, however, are seldom used; on small machines they are not required, and on those of larger size they are found to lead to trouble at the commutator through uneven distribution of the current in the different circuits.

A very full description of all different kinds of armature windings will be found in Parshall and Hobart's "Armature Windings."

§ 9. **Magnet Windings.**—The connections of the magnet windings of a dynamo are a much simpler matter than the armature windings. There are, however, several ways in which the magnet windings may be connected in circuit. Figs. 20, 21, 22, and 23 illustrate these different methods.

In Fig. 20 the source of supply for the magnet winding is independent of the dynamo ; the current is derived from some other source, a battery or another dynamo. This

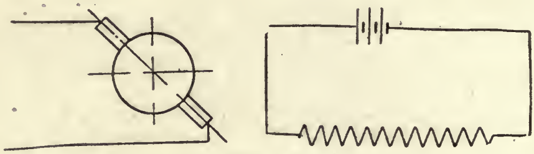


FIG. 20.

method is seldom employed in C.C. work, but alternators are almost invariably "separately excited."

Fig. 21 shows the connections for a shunt-wound dynamo. In this case the current for the field magnets is taken from

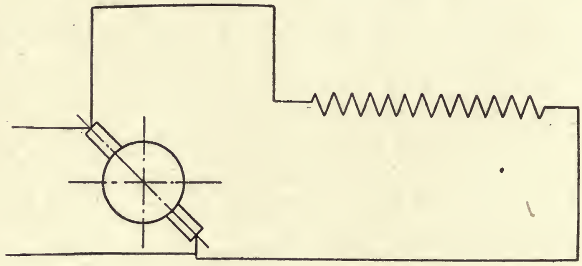


FIG. 21.

the dynamo itself, but only a small portion of the current flows through the field winding which is connected in parallel with the external circuit.

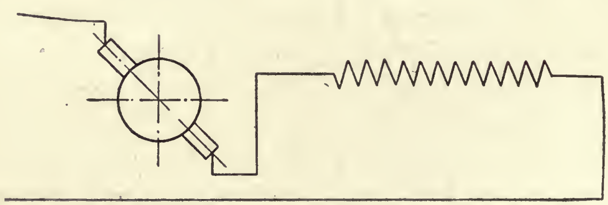


FIG. 22.

Fig. 22 gives the connections for a series-wound machine. The whole of the current is passed through the magnet winding before passing out to the external circuit.

Fig. 23 represents a compound-wound dynamo having two different circuits on the magnets, one in shunt with the external circuit, one in series with it, so that a portion

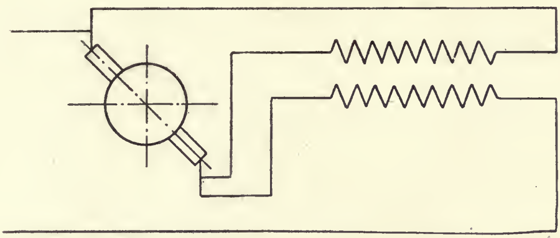


FIG. 23.

of the excitation is similar to that of a shunt-wound machine, the other portion similar to a series-wound machine.

In a shunt-wound generator the current flowing through the magnet winding depends only on the E.M.F. at the terminals of the machine, it does not directly depend on the load of the machine; this winding is therefore used for machines intended to give a constant potential; the speed being kept constant, the current in the field windings, and therefore the magnetic field, will remain approximately constant whatever the load. Inasmuch as an increase of current in the armature causes an increased drop of voltage, due to the resistance of the circuit, and causes a further diminution of the magnetic field because some of the turns of the armature winding oppose those of the magnet winding, there will be some drop in the E.M.F. generated as the load increases. This may amount to between 5 and 10%.

In the series-wound dynamo the current flowing in the magnet coils depends entirely on the load carried by the machine. The E.M.F. generated will, therefore, be small at light loads and increase as the current taken from the machine is increased. Series-wound generators are not

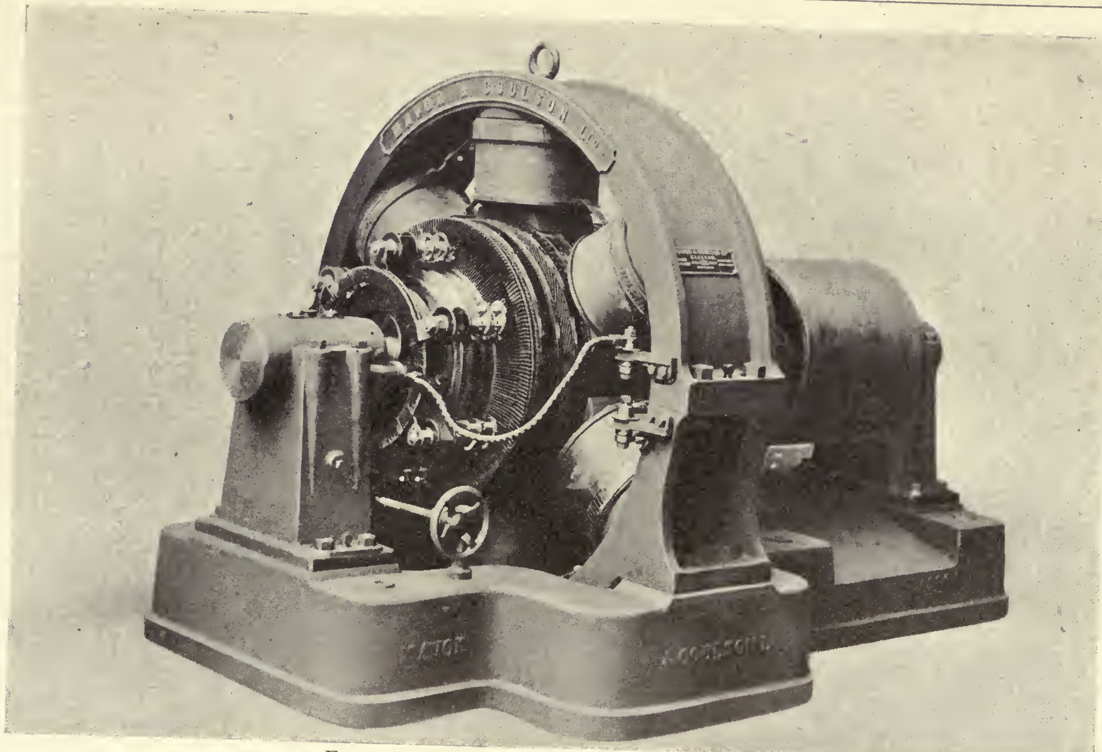


FIG. 24.—CONTINUOUS-CURRENT GENERATOR.

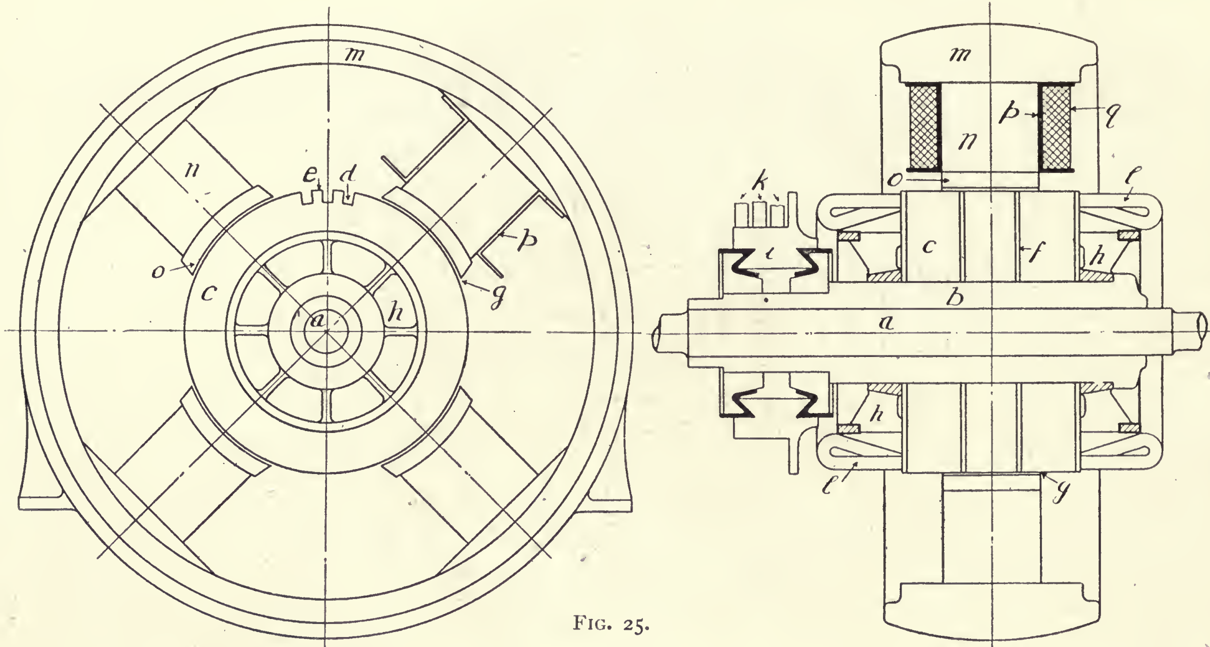


FIG. 25.

a, shaft; *b*, spider; *c*, armature core; *d*, slot; *e*, tooth; *f*, ventilating duct; *g*, air gap; *h*, end ring; *i*, commutator; *k*, brushes; *l*, end winding; *m*, magnet yoke; *n*, magnet core; *o*, pole shoe; *p*, magnet spool; *q*, magnet winding.

frequently used, but there are many cases where a series-wound motor is found useful.

The compound-wound generator is used in all cases where it is necessary that the E.M.F. should be kept constant without hand regulation. Since series turns increase the E.M.F. generated as the load increases, it is evident that the addition of a few series turns will counteract the tendency of the shunt-wound machine to drop its E.M.F. at high loads, and a machine having the correct proportion of shunt and series turns will give a constant E.M.F. whatever current is being taken from it.

It is usual to use compound-wound generators in all small installations, such as ship lighting and private house lighting. In large central stations dealing with a lighting load only, shunt-wound machines are usually put in to save the extra complications at the switchboard; the regulation of the voltage is effected by hand, by introducing or cutting out resistance in the shunt circuit.

CHAPTER III

CONTINUOUS-CURRENT GENERATOR

§ 1. Specification of Generator and Chief Dimensions.— In this chapter the calculations for the design of a continuous-current generator will be considered. The problem may be put before the designer in various ways, but a common form of occurrence is to require a generator of a stated output at a given speed. The specification in addition often demands a guaranteed temperature rise after six, ten or even more hours' run at full load. There is frequently specified in addition a given voltage regulation with various loads.

Say that it is required to design a generator to give 200 amperes, 500 volts at 420 revolutions per minute, and that the temperature is not to exceed 70° F. after a six hours' run at full load.

Let it be decided in accordance with present practice that the dynamo shall be a multipolar machine, with a slotted drum armature and a developed end winding—Fig. 24 giving the general arrangement of the machine, which is shown with six poles. The considerations on which this number is determined will be dealt with later.

It is now required to find all the principal dimensions of a machine to comply with the above requirements.

Unfortunately the conditions are too complicated to allow of any simple formula being devised in which substitution could give directly the desired result, and the process of designing any electrical machinery becomes one of trial and error. It is necessary to adopt tentatively certain dimensions, and then to calculate what may be expected of a machine of this size. To assist, however, in the first trial, previous experience must be relied on, and that is most readily done by the use of curves.

First determine the diameter and length of the armature core. A method which has been found most useful for approximating to these dimensions is based on observing the relation between the watts per revolution of the machine and the cubical contents of the armature core; if d denotes the diameter of the core and l its length parallel to the

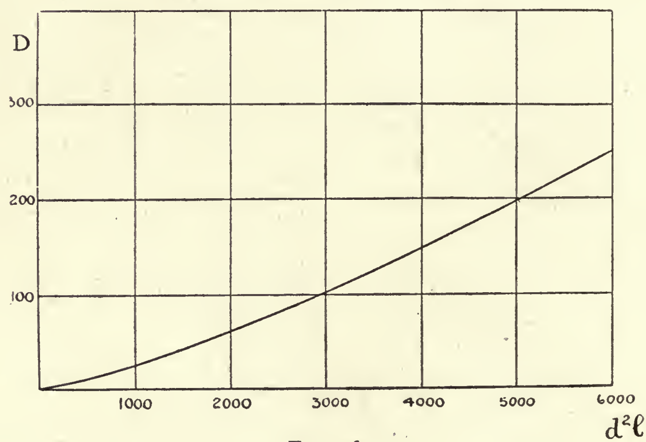


FIG. 26.

shaft, the expression $d^2 l$ is proportional to the over-all volume. Again dividing the total watts output of the generator by its speed in revolutions per minute, gives a quantity which is denoted by D and called the watts per revolution (it has been variously called by different writers the machine constant, or the output co-efficient). In the present instance the machine has to give 500 volts 200 amperes, that is 100,000 watts at a speed of 420 revolutions; therefore $D = \frac{100,000}{420} = 238$.

The curve (Fig. 26) is plotted with D as ordinates and

$d^2 l$ as abscissæ (the values of D and $d^2 l$ are calculated for a number of machines which have been found satisfactory, and a smooth curve is then drawn through the average of the points so obtained); this curve applies to well-ventilated generators of the multipolar type. Referring to this curve it is found that $D = 238$ corresponds to $d^2 l = 5{,}800''$; values of $d = 28''$ and $l = 8''$ will satisfy this. Many other values of d and l might be chosen giving the same value of $d^2 l$. The considerations which determine the ratio of l to d are that a large diameter and small length will increase the cost of manufacture, whilst a small value of l will improve the commutating qualities of the machine. As a compromise between these conflicting requirements it is proposed to adopt $28''$ as the diameter of the armature and $8''$ as the length of core.

Starting from these dimensions, it is now required to estimate the number and size of the bars on the armature, the total number of magnetic lines required from each pole in order to give the specified E.M.F., and then to calculate what the performance of such a machine may be as regards heating, sparking, and efficiency, and to see whether it will in these respects comply with the specification.

The calculations required are tabulated thus:—

E.M.F.	HEATING	SPARKING	EFFICIENCY
(1) Number of bars (2) Strength of field (3) Effect of armature reaction on strength of field (4) Voltage drop due to armature resistance	*Armature* (1) C^2R losses (2) Eddies and hysteresis in iron core *Magnets* (3) C^2R *Commutator* (4) C^2R brush contact (5) Brush friction	(1) Reactance volts (2) Effect of armature turns	(1) All losses given under heating (2) Friction of bearings and windage

§ 2. **Number of Bars on Armature.**—Take first the number of bars required on the armature. It is seen that this will depend on the total flux from the magnet and on the speed (see page 17). The speed is determined by the specification. As to the total flux, a large total number of magnetic lines per pole will mean powerful and expensive magnets, it will involve working at a high density in the air gap, and this will require more copper on the magnets and increase their cost. A high total flux will also generally mean increased losses in the armature iron.

On the other hand, a high density in the gap is good from the point of view of commutation, and the total flux must be kept high so that the number of bars on the armature does not become excessive.

A common density to work at in the air gap is from 7,000 to 8,000 lines per square centimeter. The value of B being fixed, the total flux will depend on the area of the pole shoe.

It is usual to make the length of pole shoe, measured parallel to the shaft, equal to the length of the armature core. The pole shoe area will then be equal to the length of armature core multiplied by the pole arc.

The distance from the centre of one pole to the centre of the next being called the pole pitch, the ratio of the pole arc to the pole pitch is a quantity to be carefully considered; if the pole arc is long and poles of opposite polarity are therefore close together, the amount of leakage will be greatly increased. On the other hand, too small a pole arc will reduce the pole area and therefore the number of useful lines. In a machine of this size a common ratio is ·7, in smaller machines it may fall as low as ·6. If it be agreed to adopt ·7 as the ratio of pole arc to pole pitch in this machine, the pole arc will have a length of $\frac{28\,\pi}{6}$·

$\times \cdot 7 = 10{\cdot}2''$, $28\,\pi$ being the circumference of the armature and $\frac{28\,\pi}{6}$ the pole pitch. Multiplying $10{\cdot}2''$ by $8''$ the length of armature core, $10{\cdot}2 \times 8 = 81{\cdot}6$ square inches and $81{\cdot}6 \times 6{\cdot}45 = 525$ square centimeters is obtained as the area of the pole face. But the magnetic lines from the pole shoe will spread, and there is, therefore, a fringe of magnetic lines beyond the actual area of the shoe. Take this as 5%, and 550 square cms. is the actual area effective for carrying line of force. Working at a density of 7,900 in the air gap this will give as the total number of lines $7{,}900 \times 550 = 4{,}350{,}000$.

It has been seen in Chapter I., § 8, that the E.M.F. generated in a conductor depends on the number of lines cut by the conductor in unit time. If N be the total number of lines of force from one pole of the magnet, and if there be P poles $P \times N$ will be the total number of lines of force cut by one conductor on the armature during one revolution. Again if R be the number of revolutions per minute $\frac{R}{60}$ = the number of revolutions per second, the total number of lines of force cut in one second by one conductor will be $P\,N\,\frac{R}{60}$ and the E.M.F. generated in each of the conductors will add together provided these conductors are connected in series; if therefore Z represent the number of bars on the armature connected in series, the E.M.F. generated in the armature will be

$$Z\,P\,N\,\frac{R}{60}.$$

This result gives the E.M.F. in C.G.S. units, and must be divided by 10^8 to express the E.M.F. in volts; if, therefore, E be the E.M.F. required, measured in volts,

$$E = \frac{Z\,P\,N\,R}{10^8 \times 60}.$$

For the present purpose this can best be written in the form—

$$Z = \frac{E \times 10^8 \times 60}{P N R}.$$

Substituting in this formula ; the E.M.F. required at the terminals is 500, but at full load there will be a drop due to the resistance of the armature (and of the series winding if the dynamo is compound wound). Assume this at 4%

$$500 \times \cdot 04 = 20$$

At full load, therefore, the armature must generate 520 volts, and $E = 520$, $P = 6$. N has been found to be 4,350,000

$$\therefore Z = \frac{520 \times 10^8 \times 60}{6 \times 4 \cdot 35 \times 10^6 \times 420} = 284.$$

The armature can be wound "lap" or "wave" (see Chapter II.); in the former case there will be six circuits from brush to brush in the armature, in the latter two, and the total number of bars will be 1,704 and 568 respectively. Wave winding is preferable in this instance, 1,704 bars being an excessive number, which would entail an expensive commutator and undue waste of space in insulation ; thus 568 is the total number of bars required on the armature. The number of commutator parts in a machine of this size will be half the number of bars, *i.e.*, 284, and this divided by the number of poles gives about 47 commutator parts per pole. It is well in 500-volt machines to aim at about 50 commutator parts per pole, so that in this respect the number of bars is satisfactory.

§ 3. Size of Armature Bars and their Disposal in the Slots.—The next question to consider is as to the size of the bars and their arrangements in the slots. The size of the bar depends ultimately on not losing an excessive number of watts ($C^2 \omega$) in the armature winding, ω being the resistance of the armature winding from brush to brush. As this, however, cannot be calculated at this stage, it is useful to work as a first approximation from the current density in the bars. Formerly 2,000 amperes per square inch used to be a number frequently given as a guide, but of late the usual current density has been considerably increased, and 2,000 amperes to the square inch, which used to be considered the higher limit, is probably now the lower limit of current densities in common use, the densities running from 2,000 to about 3,000 amperes per square inch.

Working at 2,100 amperes per square inch, and remembering that there are two circuits through the armature, so that each bar carries only half of the main current, the section of the bar is about ·048 of a square inch (100 amperes divided by 2,100 = ·048). Try a bar ·6″ × ·08″, 568 bars each ·6″ by ·08″ have to be disposed of in a certain number of slots. The greater the number of bars in each slot, the less will be the percentage space wasted in insulation. On the other hand, to a certain extent the winding will be less symmetrical, and if too many bars are put in one slot the result is a distinct marking of some of the commutator parts on account of this want of symmetry. The number of commutator parts connected to bars in one slot should not exceed four or five, the number of bars per slot therefore should not be more than 8 or 10. Try 8 bars per slot: 568 divided by 8 gives 71 slots, and the space required in the slot will be found as follows.

The bars will be disposed in two layers; this is necessary in order that the end windings may cross one another in two separate layers; the bars will therefore be in the slot 4, side by side, and two in the depth (see Fig. 27). The insulation consists of a layer of paper, presspahn, or similar material formed into a trough lining the slot and of a winding of cotton tape round each bundle of bars, *i.e.*, the top four are taped together and also the bottom four. The allowances made for the thickness of these various insulations is ·02″ for tape on the separate bars, ·03″ for the tape enclosing

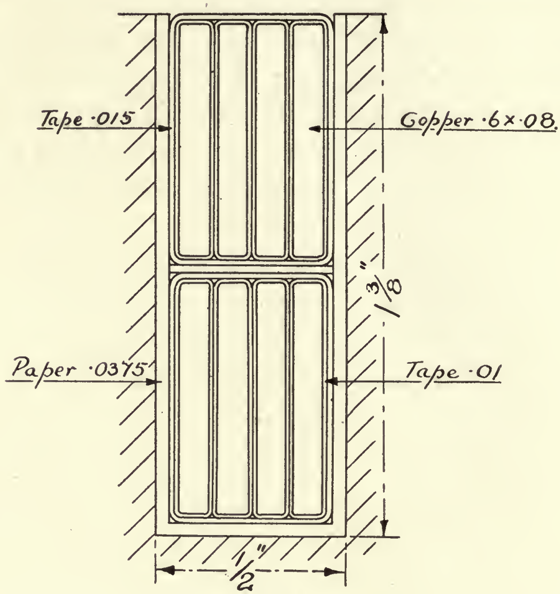


FIG 27.

the bundle of bars, ·055″ for the slot insulation. In addition to this some allowance for slack must be made, and thus are obtained :

	DEPTH		WIDTH
Copper and tape .	·6 + ·02 2 =	1·24	(·08 + ·02) 4 = ·4
Tape round bundle .	·03 × 2 =	·06	·03
Paper .		·055	·055
Slack .		·02	·015
		1·375 = $1\frac{3}{8}''$	·500 = $\frac{1}{2}''$

These allowances are clearly indicated in Fig. 27.

Equally spaced on an armature 28″ diameter, 71 slots will give 1·24″ as the slot pitch $\left(\frac{28\,\pi}{71} = 1{\cdot}24\right)$ the distance from centre of one slot to centre of next, and subtracting from this ·5″, the width of the slot ·74″ is left as the width of one tooth. The more important point, however, is the width of the tooth at the bottom of the slot. It is there at its narrowest, and the area at the bottom therefore determines the maximum number of magnetic lines which can be put through (see Fig. 28). Subtract the depth of two slots from the diameter of the armature, this gives the diameter at the bottom of the slots; in this case it

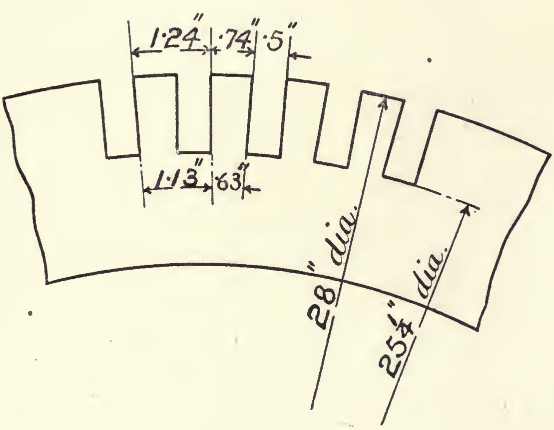


FIG. 28.

is 25¼″. Proceeding as before, the tooth pitch at this diameter is $\frac{25\frac{1}{4}\,\pi}{71} = 1{\cdot}13''$ and again subtracting ·5, the width of the slot, ·63″ is obtained as the width of the tooth at its narrowest part.

To find from this the area of the tooth available for carrying magnetic lines, first determine the effective length of the iron in the armature core measured parallel to the shaft. In Chapter II. it has been pointed out that the armature core is built up of thin iron discs, which are insulated from one another. At intervals along the length of the armature, it is usual to put in distance pieces, thus leaving ducts in the core, through which air is thrown out for cooling purposes. The effective length of the armature will therefore be the gross length less the width of these ventilating ducts and less the space taken up by insulation between the individual discs. It may be arranged to put in three ventilating ducts each half an inch wide; this will give a total duct width of $1\frac{1}{2}''$, which, subtracted from $8''$ leaves $6\frac{1}{2}''$ of discs and insulation. The insulation between discs is usually taken to amount to about 10%, and the effective length of iron in armature $6{\cdot}5 \times {\cdot}9 = 5{\cdot}8''$ is thus found.

The number of teeth under one pole will be 71 divided by 6 and multiplied by ·7, the ratio of pole arc to pole pitch, which is equal to 8·3. An allowance for the fringe must, however, be made. Assume this fringe at 10%, the number of teeth carrying the flux from one pole will therefore be $8{\cdot}3 \times 1{\cdot}1 = 9{\cdot}1$, and the area available at the bottom of teeth will be $9{\cdot}1 \times 5{\cdot}8 \times {\cdot}63 = 33{\cdot}5$ square inches. Multiply this by 6·45 to make it into square centimeters, and divide the total number of lines per pole, namely 4·350,000, by this area.

$$\frac{4{,}350{,}000}{33{\cdot}5 \times 6{\cdot}45} = 20{,}000 \text{ lines per square centimeter.}$$

This is quite a safe value for the induction at the bottom of the teeth. Good limiting values for this number are 20,000 to 22,000. The considerations which determine this are that a high value gives what is known as a stiff field; that is, the iron of the teeth being strongly saturated, the strength of field will not readily be affected by the armature current; whilst, on the other hand, with a higher value of tooth induction the number of ampere-turns required on the magnets to overcome the reluctance of the highly saturated iron rapidly becomes excessive.

§ 4. **Armature Heating.**—Having determined the size and number of bars and their arrangement in the slot, the next point will be to find how many watts are being lost on the armature. These will consist of the losses due to the copper resistance and those due to hysteresis and eddies in the iron. Take the copper losses first. The resistance of a conductor varies directly as its length, and inversely as its section.

To calculate the length of a bar, assume the cylindrical surface of the armature developed into a plane, the coil will then appear as in Fig. 29. Each coil is made up of two bars, such as A D lying in the slot, and of end connections such as B C and D E; E and F are connected to the commutator. The length of one bar and its end connection from C to E is the length required. The length of the bar in the slot will be 8″, but as the bend must not be too sharp, add half an inch at each end, making the total length of straight bar A D = 9″. The length of end plate A B required is approximately obtained by dividing the diameter of the armature by the number of poles, in this case about $\frac{28}{6} = 4{\cdot}6''$; B C is the length measured along the armature periphery taken up by one bar; this length is obtained by dividing the pole pitch by 2, that is, $\frac{1}{2} \times \frac{28\,\pi}{6} = 7{\cdot}2$ but $AC = \sqrt{AB^2 + BC^2} = \sqrt{(4{\cdot}6)^2 + (7{\cdot}2)^2}$ $= 8{\cdot}5$. The total length of a bar with its end connections

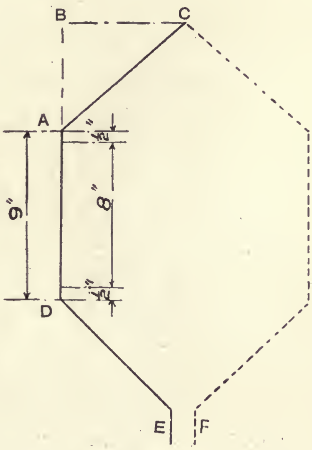


FIG. 29.

will therefore be DB + AC + DE + allowance for bend at C, say, $9'' + 8\frac{1}{2}'' + 8\frac{1}{2}'' + 2'' = 28''$; but there are 568 such bars, and the total length of copper will therefore be

$$568 \times \frac{28}{12} = 1{,}300 \text{ feet.}$$

The resistance of the copper can be found either from the known resistance of a cubic inch, or, as is perhaps more convenient in practice, from a table, giving the resistance per foot of different sections. A sample of such a table is given in the Appendix. From such a table, the section being ·6 by ·08, the resistance is found to be ·00019 ohm per foot, and therefore ·246 ohm for 1,300 feet. This resistance includes 12% for the increased resistance due to the heating of the armature. In proceeding to calculate the watts lost, it must be remembered that there are two paths through the winding which are connected in parallel by the brushes, and that therefore the resistance obtained for the whole length of winding must be divided by 4 to give the resistance from brush to brush. The $C^2 \omega$ losses are therefore

$$\frac{\cdot 246}{4} = \cdot 0615 \text{ ohm resistance from brush to brush,}$$

$\cdot 0615 \times 200 = 12\cdot 3$ volts dropped,

$12\cdot 3 \times 200 = 2460$ watts lost.

The losses in the iron are usually found by working out the losses per pound and multiplying by the total weight. These losses depend upon the induction, that is the number of lines per square centimeter, and also on the frequency. It can be shown quite readily that the eddy losses vary as the square of the periodicity and as the square of the induction. The hysteresis losses vary directly as the periodicity; they increase with, but more rapidly than, the induction, and are usually assumed to vary as $B^{1\cdot 6}$. From this it follows that as the induction in different parts of the circuit will be different, being for instance much higher at the bottom of the tooth than in the body of the core, these different parts should really be separated and calculated each by itself. In practice, however, it is found sufficiently accurate to work from a curve giving the relation between periodicity multiplied by B and watts lost per pound. Such a curve is given below (Fig. 30). The induction B in the core, and the number of reversals per seconds ascertained, these numbers are multiplied together, and from the curve the corresponding number of watts lost per pound of iron is read off.

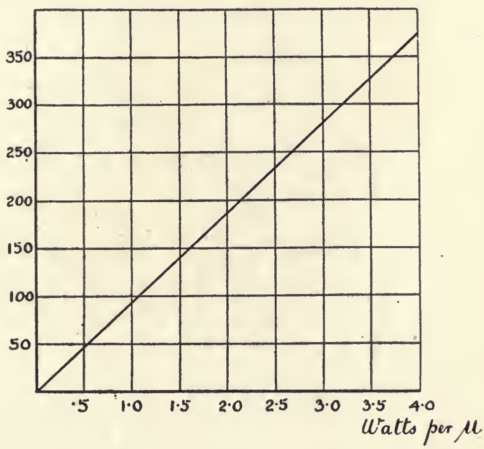


FIG. 30.

In order to make some allowance for the higher inductions in the teeth, it is usual to calculate the weight of the core as if there were no slots. The small extra weight thus gained makes some allowance in the calculation for the fact that the losses in the teeth will be greater than calculated on. The curve is based on the assumption that the induction in the core does not differ very greatly from 10,000 lines per square centimeter.

Now must be calculated the depth for the core discs, giving the diameter of the hole in the centre of the discs.

The effective length of iron has already been found to be 5·8″, and in order to work at about 10,000 lines per square centimeter in the armature core the sectional area must be

$\frac{4 \cdot 35 \times 10^6}{10,000} = 435$ square centimeters. Note, however, that the lines from one pole on passing into the core split up into two paths, half of them passing to the adjacent pole on the right hand, and half to the adjacent pole on the left hand (Fig. 31), and at any one section of the armature

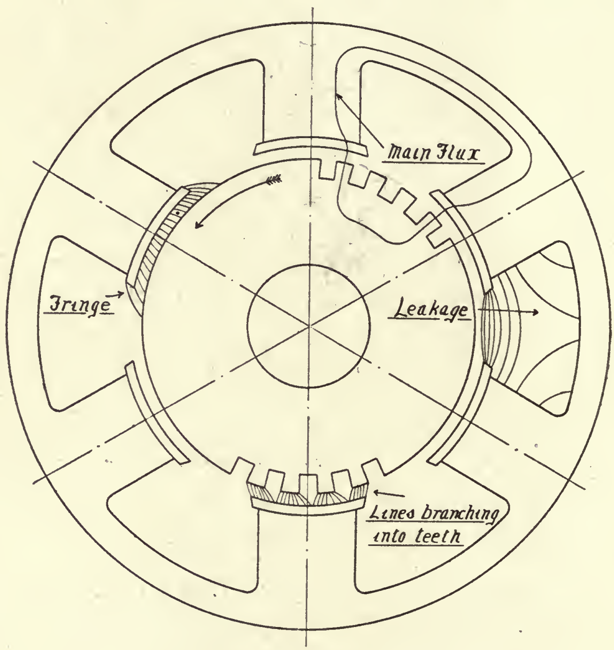


FIG. 31.

core it need only be large enough to carry half of the total number of lines, *i.e.*, it need only be 218 square centimeters in section. But this is equal to $\frac{218}{6 \cdot 45} = 34 \cdot 5$ square inches, and since the effective length is $5 \cdot 8''$, the depth of the core from the bottom of the slot to the hole in the centre will be $\frac{3 \cdot 45}{5 \cdot 8} = 6''$. To allow a little margin and to bring the diameter of the centre hole into round figures call this $6\frac{3}{8}''$.

Adding to this the depth of the slot, $7\frac{3}{4}''$ is obtained, and multiplying by 2 and subtracting from 28, gives for the diameter of the hole in the centre $12\frac{1}{2}''$ (see Fig. 32). To obtain the weight of iron in this core, the equation $\frac{\pi}{4}(D^2 - d^2)\, l \times \cdot 28$ may be used. Where D is the

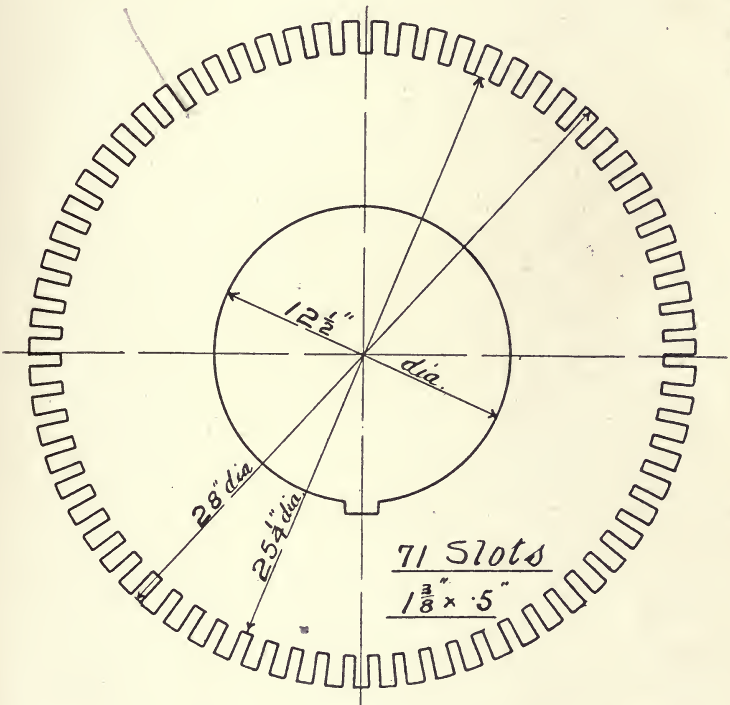


FIG. 32.

external diameter of the armature, in this case 28″, d is the internal diameter $12\frac{1}{2}''$ and l is the effective length of iron; all these dimensions being in inches, multiply by ·28, which is approximately the weight in pounds of a cubic inch of iron. Note that $\pi\left\{\left(\frac{D}{2}\right)^2-\left(\frac{d}{2}\right)^2\right\}$ is the area of the annular space occupied by the discs; multiplying by

l gives the volume. Substituting the already known values in the above equation

$$\frac{\pi}{4}\left((28)^2 - (12\tfrac{1}{2})^2\right) 5\cdot8 \times \cdot28 = 850 \text{ lbs.}$$

The number of reversals per second is equal to the revolutions per second multiplied by the number of pairs of poles $\frac{420}{60} \times 3 = 21$, and the induction B in the core is 10,000 lines per square centimeter; multiply these together $21 \times 10{,}000 = 210{,}000$, and from the curve (Fig. 30) the corresponding watts per lb. are found to be 2·2. The total watts are therefore $2\cdot2 \times 850 = 1{,}870$.

Having thus found separately the watts lost in the copper and in the iron, these, added together, give the total watts $1{,}870 + 2{,}460 = 4{,}330$ expended in heating the armature, and the heat so generated must be given off from the surface of the armature, and this radiating surface is $28\,\pi \times 18\cdot25 = 1{,}600$ square inches. It will be seen from Fig. 29 that $18\frac{1}{4}$ is the length of armature measured over the windings. Dividing the watts by this

$$\frac{4{,}330}{1{,}600} = 2\cdot7 \text{ watts per square inch.}$$

What amount of heating may be expected from this number of watts per square inch is a matter of experience on different types of machines. There is given below a curve (Fig. 33) showing the number of degrees Fahrenheit rise above the surrounding atmosphere which may be expected from the loss of one watt per square inch in an open type machine for different peripheral speeds; the higher the peripheral speed the greater the cooling effect due to the circulation of air. The peripheral speed of the present armature is $\frac{28}{12} \times 420 = 3{,}150$ feet per minute, and from this it would appear that the armature, as it has now been wound, will have a temperature rise of about $26 \times 2{\cdot}7 = 70°$ F., and is sufficiently safe as regards heating. It must, however, be tested also from the point of view of sparking. This will be deferred, and the design of the field magnets suitable for such an armature be first dealt with. The lines on which this may be done are as follows.

§ 5. **Dimensions of Magnet.**—The total number of lines per pole which must be put into the armature has already

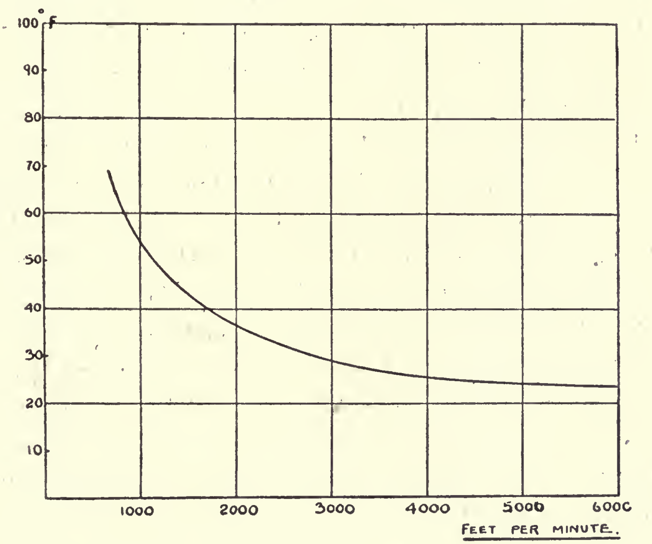


FIG. 33.

been found, the section of metal on the different parts of the magnets required to carry that number of lines must now be ascertained, after which the next step will be to find the number of ampere-turns required to put this total flux through each part of the magnetic circuit.

First of all the parts to be considered will be the air-gap, the teeth, and the armature core, and then the magnet core and the magnet yoke, which are frequently of different sections.

Magnets are now usually made of cast steel. The permeability does not differ materially from that of wrought iron. The material, however, is somewhat less trustworthy; the permeability in different specimens varies largely, and it is therefore not safe to work at a very high induction; B = 13,000 to 13,500 per square centimeter is a safe value to take. It must also be noted that higher values of the induction mean a greater number of ampere-turns on the magnets, and this, as will be seen, means increased weight of copper, so that the saving in steel effected by reducing the section is lost in the increased cost of copper required on the magnets.

The total number of lines per pole is 4,350,000, but this number represents the useful lines which actually cut the armature conductors, and there will be in addition to this a certain number of lines passing through the magnet coil, and therefore through the steel of the magnet core, which "leak" through various paths, and do not enter the armature. This leakage is clearly shown in Fig. 31, and to allow for it the number of useful lines is multiplied by a "leakage coefficient." A safe value to take in such a machine as we are considering is 1·2. The total number of lines passing through the magnet core and yoke will then be $4,350,000 \times 1{\cdot}2 = 5,200,000$, and in order to work at a density of 13,000 lines per square centimeter, the area of the pole will have to be—

$$\frac{5,200,000}{13,000} = 395 \text{ square cms.}; \quad \frac{395}{6{\cdot}45} = 61{\cdot}5 \text{ square inches.}$$

A good section for the magnet cores is to make them circular; for a given area this gives the smallest length of periphery, and thus shortens the length of copper wire required for the winding. If the magnet cores be made $8\frac{3}{4}''$ diameter, this gives a sectional area of 61·5 square inches. The area of the ring carrying the magnet cores must also be 61 square inches or thereabouts; but here again, as in the case of the armature core, the lines from one pole have two paths through the ring, and since half of them only pass through any given section, the section can therefore be reduced to 30·5 square inches. Fig. 31 shows the magnetic circuit of the machine and the approximate paths of the lines of magnetic induction. At this stage it is advisable to make such a drawing to a fairly large scale.

The length of the magnet core is determined by the space required for the magnet winding; in this case 7″ will be enough between the flanges; $1\frac{1}{2}''$ for the pole shoe, and about 1″ to allow for the curvature of the ring (Fig. 36), giving a total length from the face of the pole shoe to the inside of the magnet ring of 9″. The depth of air gap, that is the distance from the top of the teeth to the pole face, has been taken at $\frac{1}{4}''$. The greater this dimension, the greater will be the number of ampere-turns required to get the magnetic flux through the air space; on the other hand, enough space must be left for mechanical clearance, and also to insure that the armature ampere-turns shall not be too powerful in comparison to the magnet turns.

§ 6. Calculation of Ampere-turns required on the Magnets.—A table is given below showing the calculation of the ampere-turns required on each part of the magnetic circuit.

The first column gives the area in square centimeters the second column gives the magnetic induction B, which is in each case obtained by dividing 4,350,000, the number of lines per pole, by the sectional area given in column 1. Column 3 is the value of H corresponding to B; in the case of air H is numerically equal to B; in the case of iron or steel it is obtained from the magnetisation curves given in Fig. 5. Column 4 gives the length in centimeters of the path of magnetic lines; where not already ascertained this is got by scaling off the drawing (Fig. 31); column 5 is obtained by multiplying together the values in column 4 and column 3; column 6 gives the number of ampere-turns required, and is obtained from column 5 by multiplying by $\frac{10}{4\pi} = \cdot 8$ sufficiently nearly (see Chapter I.).

	AREA	B	H	l	H × l	AMPERE-TURNS
Air gap .	551	7,900	7,900	$\frac{1}{4}''$ = ·625	5,100	{ 4,080 400
Arm core	430	10,000	2	$4\frac{1}{4}''$ = 11	22	17
Teeth top	241	17,600	} 145	$1\frac{3}{8}''$ = 3·45	420	336
Teeth root	205	21,000				
Mag. core	395	10,800 × 1·2 = 13,000	16	9″ = 23	370	296
Mag. ring	395	10,800 × 1·2 = 13,000	16	12″ = 30	480	384
						5,513

The only explanations required to make this table clear relate to the air gap and to the teeth. It is evident from inspection of Fig. 31 that the magnetic lines in passing across the air gap will not be uniformly distributed, and will tend to gather into bunches at the top of the teeth. The effective area of the air gap which actually carries lines, will therefore be less than the pole-shoe area, and on this account the number of ampere-turns required will be somewhat increased. This point has been discussed at great length by Professor Hele-Shaw in a paper read before the Institution of Electrical Engineers.*

The other point is the value of H to be taken for the teeth. The induction in the teeth evidently varies from

* See *Journal* of the Institution of Electrical Engineers, No. 170, Vol. XXXIV.

the top to the bottom with the varying section, and the question as to what is the value of the magnetic force H is one of considerable complexity. This question has also been treated of in Professor Hele-Shaw's paper referred to above. As, however, the full consideration of both these points is too advanced for an elementary treatise, it has been considered sufficient in the present instance to add about 10 per cent. to the ampere-turns required for the air gap as an allowance for the bunching of the lines at the top of the teeth. In the case of the teeth the mean induction B has been calculated, and the corresponding value of H taken from the curve in Fig. 6.

$$\text{Average B in teeth} = \frac{21{,}000 + 17{,}600}{2} = 19{,}300.$$

From the above table the total number of ampere-turns required to put the flux through the magnetic circuit is ascertained to be 5,500; in addition to this provision must be made for the demagnetising turns on the armature.

If the brushes on the commutator are exactly on the neutral line, there will be no back ampere-turns, the whole effect of the armature winding will be what is known as cross turns. If, however, as is usually the case, the brushes are not on the neutral line, the turns on the armature can be divided into cross turns and back turns. This can readily be seen from Figs. 34 and 35. In these figures a dot indicates a current flowing towards the reader,

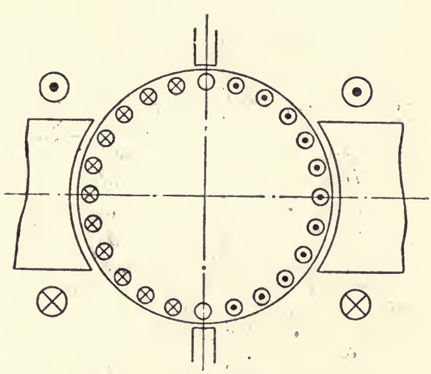


FIG. 34.

a cross a current flowing away from him. In Fig. 34, where the brushes are on the neutral axis, the number of dots and crosses on any one pole gap are equal; the currents therefore neutralise one another. In Fig. 35, where the brushes are displaced, there are more currents flowing in one direction than in the other, and it will be seen that in each case the excess current flows in such a direction as to oppose the magnet current. The amount of back ampere-turns is proportional to the angle included by the lines *a* and *b*, that is, to twice the angle through which the brush is displaced. As the exact effect of these turns depends on the brush position, which is not easily determined beforehand, it is usual to take the total number of ampere-turns per pole on the armature, and allow a certain percentage of these, say 30 to 35 %, as being required on the magnets to compensate for armature disturbance.

In the present instance there are 568 bars or 284 turns, each carrying one hundred amperes, that is, on the armature there are 284 × 100 ampere-turns; dividing this by 6 gives 4,730 armature ampere-turns per pole. Take 35 % of these; this will give an additional 1,680 ampere-turns required on the magnets. That is, altogether there will be required 5,500 + 1,680 ampere-turns, say 7,200 per pole. A suitable winding must now be found for the magnets, so that they shall have this number of ampere-turns.

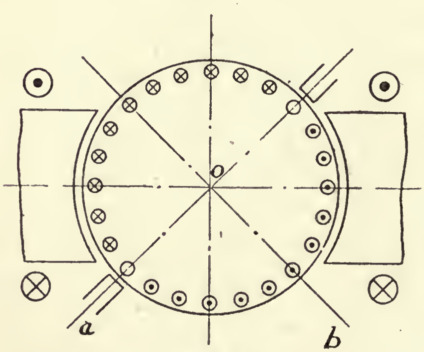


FIG. 35.

§ 7. **Magnet Winding.**—The diameter of the core has been found to be $8\frac{3}{4}''$; insulation will have to be provided between the steel of the magnet and copper winding. In a machine of this size it is usual to put the winding not directly on to the core, but on a former which must be of dimensions suitable for slipping on to the core, taking into account possible inequalities of the casting. This former is sometimes made of sheet iron, in which case it is necessary to allow for a layer of paper, cloth, or other insulating medium before the wire is put on; in other cases the former is made of some insulating substance in the nature of papier mâché, or one of the many similar substances on the market.

In the latter case the dimensions of the former can be worked from directly. In order to slip over a magnet core $8\frac{3}{4}''$ in diameter, the inside of the former should be not less than about $9''$, and allowing $\frac{1}{8}''$ for the insulating material, whether that be the former itself or a separate layer of insulation, this gives the total of $9\frac{1}{4}''$ as the diameter on which wire has to be wound. The length of the winding space required will not be far off $7''$ in this machine. If it be made shorter, the depth of winding must be increased in order to get in a sufficient quantity of copper, and it is advisable that the depth of winding should not become too great, as the heat is then retained and the cooling is much less satisfactory.

Suppose that $7''$ be tried as the length between the cheeks of the former, and as a first assumption let it be assumed that $2''$ is the depth of copper wire which will be required. The length of the mean turn will then be obtained as follows. The shortest turn is wound, as has been seen, on a diameter of $9\frac{1}{4}''$. The outer turn will have a diameter of $9\frac{1}{4}''$ plus twice $2''$, that is $13\frac{1}{4}''$. The mean turn, that is the turn having the average length, will then be wound on a diameter of $11\frac{1}{4}''$, since this is the mean between $9\frac{1}{4}''$ and $13\frac{1}{4}''$, and the length of the mean turn will thus be $11\frac{1}{4} \times \pi$, which equals $35{\cdot}5''$.

The next thing is to ascertain how many watts may be got rid of in such a winding. If the cylindrical surface of the coil be calculated on as being the only cooling surface, it may reasonably be expected to get rid of about ·7 to ·8 watt per square inch of this surface. The outside turn will be wound on a diameter of $13\frac{1}{4}''$ and the circumference of the coil on the outside will therefore be $13\frac{1}{4} \times \pi = 42''$, and the length between the cheeks will be $7''$. The area of the cylindrical surface is therefore $42 \times 7 = 294$ square inches, and it being expected to radiate ·72 watt from each square inch, the total watts which may be lost in one coil will work out at $294 \times {\cdot}72 = 210$ or 1,260 on the six coils.

The E.M.F. of the machine is 500 volts, and therefore there will be on each coil, the six magnet coils being connected in series, an E.M.F. of $\frac{500}{6} = 83$ volts. To give a margin allowing of the use of a shunt resistance, take 20% off this, and take 65 volts as the E.M.F. on each coil. The current is then obtained by dividing the watts lost by the E.M.F. $\frac{210}{65} = 3{\cdot}3$ amperes, and the number of turns of wire required is obtained by dividing the ampere-turns required by the current $\frac{7{,}200}{3{\cdot}3} = 2{,}170$. There are therefore required 2,170 turns of wire on each coil, and since the length of the mean turn is $35{\cdot}5''$ the length of wire will be—

$$\frac{35{\cdot}5 \times 2{,}170}{12} = 6{,}440 \text{ ft.}$$

Again, if the E.M.F. on each coil is 65 volts, and the current is to be 3·3 amperes, the resistance must be

$\frac{65}{3 \cdot 3} = 19 \cdot 8$ ohms. It is therefore required to find a suitable wire, so that the resistance of 6,440 ft. shall be 19·8 ohms. This can be got most easily from some of the many published tables of the properties of copper wire. (See sample table in Appendix.) Find in such a table a size of wire the resistance of which is $\frac{19 \cdot 8}{6{,}440} = \cdot 0031$ ohm per ft. Number 16 S.W.G. comes very near this requirement, the resistance being ·003 ohm per ft.

Notice that in the case of a shunt winding, the size of the wire alone determines the number of ampere-turns. This is on the assumption that the length of mean turn remains unaltered, and is evident on the following consideration. If, having wound the coil with a certain number of turns, this number is now doubled the resistance is also doubled, and therefore half the current will flow in the coil. The result of doubling the turns is therefore to have half the current flowing through twice the number of turns, leaving the ampere-turns unaltered. The gauge of wire used in shunt winding, therefore, practically determines the number of ampere-turns, and the number of turns of such a wire which must be used is determined simply by the number of watts which may be lost. That is to say, the heating of the coil depends on the number of turns which are put on; the ampere-turns depend on the size of wire used.

On further reference to the copper table it is found that the diameter of No. 16 copper wire is ·064″. The wire will be double cotton-covered, and the thickness of this covering may be taken as giving an increase of ·012″ in the over-all diameter. The diameter over the insulation will therefore be ·076″, and the length between the flanges of the former has been taken as 7″. The number of turns which can be got in a layer will therefore be 7″ divided by ·076 = 93. Some slack in the winding must, however, be allowed for. With careful winding 10% should suffice for this. This will reduce the number of turns per layer to 84. If there be 84 turns per layer, and 2,170 turns required, the number of layers will be $\frac{2,170}{84} = 26$, and the depth taken up by 26 layers will be $\cdot 076 \times 26 = 1\cdot 96''$. Add 10% to this, which will give nearly 2·15″ as the depth of the coil. This depth is sufficiently near the 2 originally assumed, and the coil may be wound with 26 layers, 84 turns per layer, of No. 16 S.W.G. wire. The length of this will be—

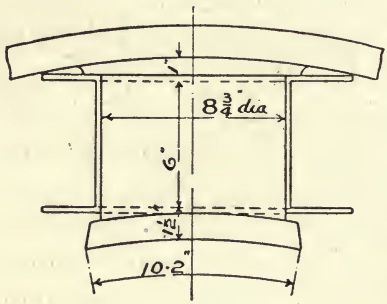


FIG. 36.

$$\frac{26 \times 84 \times 35\cdot 5}{12} = 6,500 \text{ ft.}$$

From the copper wire table the weight of No. 16 wire is found to be ·0124 lb. per ft., and the weight of copper on one coil will therefore be $6,500 \times \cdot 0124 = 80$ lbs.; on the whole machine $80 \times 6 = 480$ lbs. The resistance of one coil hot will be $6,500 \times \cdot 003 = 19\cdot 5$ ohms, and if 15% be taken from this about 16 ohms is obtained as the resistance cold; for the six coils in the machine connected in series the resistance will be $19\cdot 5 \times 6 = 117$ ohms hot, and 96 ohms cold. The current flowing through the shunt circuit will be $\frac{500}{117} = 4\cdot 2$ and $\frac{500}{96} = 5\cdot 2$ respectively, and the number of ampere-turns on each coil obtained by multiplying the current by the number of turns will be

2,170 × 4·2 = 9,000; this allows of ample margin over the 7,200 ampere-turns we calculated as being necessary, and a shunt-regulating resistance connected in series with the magnet winding will allow for regulation of the voltage and also compensate for the extra current which flows through the windings when they are cold and their resistance is therefore lower than calculated.

If it be required that the machine should not be shunt wound, but compound wound, it is necessary to distinguish in working out the number of ampere-turns between the ampere-turns required at no load and those required at full load. This can be done by calculating again the value of N, noting that at no load there is no drop in the armature, and that the value to be substituted in the E.M.F. formula is therefore in this case 500 instead of 520. Then again work out the table of inductions and corresponding ampere-turns for this new value of N; at no load there is of course no armature reaction. Proceeding on these lines it will be found that the ampere-turns required at no load are 4,900.

Proceed in exactly the same way as above to find a wire which will give 4,900 ampere-turns for the shunt winding. The extra 2,300 ampere-turns must be provided for by the series winding. The total current of the machine at full load is 200 amperes; in order that this should give 2,300 ampere-turns per coil, there must be on each coil 11 turns. The easiest way of finding the section of copper required is to start from the current density. The current density on a machine of this size is usually from 800 to 1,000 amperes per square inch. Assuming 900, a section of ·22 square inch will be required. Wind with copper 1″ by ·22″, the length of this copper per coil will be $\frac{37 \times 11}{12}$ = 34 ft., and allowing for the length required for connection from one coil to the next, say 35 ft., the resistance of this will be $\frac{35 \times \cdot 0000095}{2} = \cdot 00166$ (·0000095 is the resistance of a piece of copper 1 ft. long and 1 square inch in section). The watts lost on each coil are obtained by multiplying this resistance by the square of the current, that is $\cdot 00166 \times 40{,}000 = 67$; this number must be added to the watts lost in the shunt winding in calculating the watts lost per square inch.

§ 8. **Commutation.**—The winding of both the armature and of the magnet coils has so far been considered and calculated merely from the point of view of heating. It is now necessary to look at what may be expected from the machine from the point of view of commutation. The

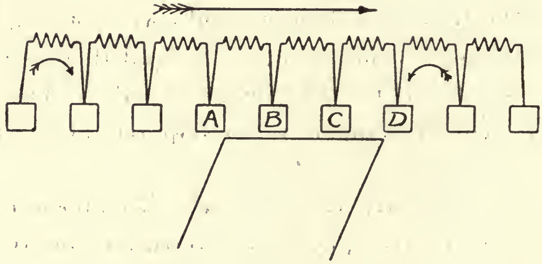


FIG. 37.

commutator consists of separately-insulated copper laminæ to which the armature windings are connected, and it is evident (see Fig. 37) that as the commutator parts come under the brush, one or more of the sections of the armature will be short-circuited for the moment. During that interval the current in the short-circuited coil must fall in value from its maximum to zero, and again rise to its maximum value in the opposite direction. If it fails to attain the correct value at the instant the commutator part leaves the brush, the current in the coil must be forcibly brought to this value by the mechanical action of the brush, and it is this which causes sparking.

From the figure again, it can be seen that as the commutator parts approach the brush, the coils as shown on the left-hand side of the diagram have the current circulating round in a clockwise direction. As they pass the brush the coil is momentarily short-circuited, it then leaves the brush, and passes into the right-hand half of the figure, where the current is circulating round the coils in a counter-clockwise direction. If the current has approximately accomplished this reversal before leaving the brush, the commutation is good, but, if it has failed to do so, the current must suddenly be brought to the right value, and this will be accompanied with sparking more or less violent as the reversal has been less or more nearly accomplished. It is therefore most important to consider what is happening during the period of short circuit.

If there were no self-induction in the coil, that is, if there were no tendency for the current to keep on flowing in the same direction, it is evident that the mechanical action of the brush would in itself be sufficient to give the current its proper value. The changing contact resistance between the brush and the commutator segments would cause more of the current to flow into the segment A (see Fig. 37), as the surface of contact between this segment and the brush increases, whilst the opposite effect would occur in segment D, which is leaving the brush, so that as the contact surface between these diminishes the resistance would gradually increase, and less of the current would therefore flow through commutator segment D, the current which was flowing through this segment being gradually diverted through the coil D C, and flowing into the brush through segment C, thus gradually changing the direction of the current through coil C D, so as to bring it into the same direction as that in the armature coils on the right of the figure.

However, the E.M.F. due to self-induction must be considered. The tendency of this is always to oppose any change in the value of the current, and it will therefore have the effect of making the reversal incomplete by the time the segment is ready to leave the brush.

This E.M.F. of self-induction may be compensated for by the E.M.F. due to the motion of the armature conductors through the main magnetic field. In the case of a generator the field magnet which the conductors have just left is of such a polarity as to give an E.M.F. in the same direction as the current flowing before the short circuit. It is therefore under the other pole, that which the bars are approaching, that a suitable field must be looked for to assist in reversing the current. Unfortunately, this is the pole tip which is weakened by the cross turns on the armature, and therefore only a comparatively weak reversing action is obtained; whilst the fact of having to move the brush forward in order to find such a field is harmful by increasing the number of back ampere-turns on the armature (see § 6, page 70).

When copper brushes were the only ones in use, these considerations were of great importance. It was always necessary to move the brushes with change of load in order to find a sparkless position. Now, however, carbon brushes are invariably used, and it is quite usual to specify that they shall run sparklessly with any load and without change of position. This is possible partly because the carbon brush, having a much greater contact resistance with the copper segments of the commutator, has an increased effect in reversing the current, but also to a great extent because the sparking between the carbon and copper is not nearly so detrimental as that between copper and copper, and that therefore a slight amount of sparking, which is practically invisible, may take place with a carbon brush without any injurious result, whilst the same amount occurring on a copper brush would spoil the commutator surface, which would have the effect of causing the sparking to become more and more violent. In spite of these advantages of the carbon brush, it is necessary, in order to secure sparkless running with fixed brush position, to take special precautions to keep the self-induction of the armature coil as low as possible, since the possibility of counteracting it by means of a suitable value of the main field is excluded by the specified fixed position of the brush.

The effect of the self-induction of the short-circuited coil is estimated by calculating the reactance voltage of the coil, that is the E.M.F. generated in the coil by variations in the magnetic flux due to the current in the coil itself. For a fuller explanation of reactance voltage, see Chapter VII.

The reactance voltage as calculated by the formulæ given below should, if possible, be under two volts if the dynamo is to run at all loads with a fixed brush position. In the larger machines it is impossible to obtain so low a value except by using very abnormal dimensions for the armature core and, provided care is taken that the other constants of the machine are suitable, it is quite possible to obtain satisfactory designs in which the value of the reactance voltage is considerably greater than two volts. The safest course, however, is always to keep the reactance voltage as low as practicable.

The formula for calculating the reactance voltage was first published in Parshall and Hobart's book on Dynamo Design; they calculate the E.M.F. due to the self-induction of the short-circuited turn on the assumption that, in a slotted armature of ordinary type, a current of one ampere flowing through one turn will give rise to twenty magnetic lines for every inch length of core. This formula has since been modified by Hobart, who, instead of using twenty lines per inch, distinguishes between that part of the wire embedded in the slots and the part which is used in the end connections. For simplicity, however, using the original method, their calculations run as follows :—

The length of core is 8″, and there is one turn per section. If one ampere were flowing through the winding one turn would enclose $8 \times 20 \times 1 = 160$ lines. Each commutator section including insulation is $\frac{1}{4}''$ wide, and each brush is $\frac{3}{4}''$ wide; these dimensions of commutator and brushes are worked out in § 9 (see page 83) of this chapter. The largest number of coils short-circuited at one time under one brush is therefore 3 (see Fig. 38), but three other coils short-circuited under the brush of opposite polarity lie in the same slots, and the current is therefore being reversed in six coils simultaneously; the number of lines due to one ampere in these six coils is therefore $160 \times 6 = 960$. Multiply this by 10^8 to change from C.G.S. units to volts and there is obtained as the value of L, the coefficient of self-induction, $L = \cdot 0000096$. Assuming the change of current to follow the sine law, the reactance of the turn will be $\pi\, n\, L$, where n is the number of reversals per second.

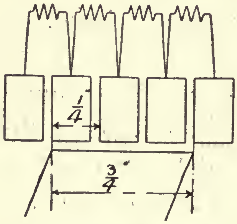


FIG. 38.

The revolutions per second are $\frac{420}{60} = 7$, the time of one revolution is therefore $\frac{1}{7}$ second, and the periphery of the commutator is $21\,\pi = 66''$, whilst the brush width is $\frac{3}{4}''$; the time required for one coil to pass under the brush is therefore $\frac{\cdot 75}{66}$ of the time required for one revolution, $\frac{1}{7} \times \frac{\cdot 75}{66} = \frac{1}{583}$ and the number of reversals per second is therefore 583. Substituting these figures, there is obtained for the reactance of the coil $\pi \times 583 \times \cdot 0000096 = \cdot 0174$, and the reactance voltage is obtained by multiplying this by the current flowing in each bar, 100 amperes—

$$\cdot 0174 \times 100 = 1\cdot 7.$$

This formula cannot be used until all the details of the design have been worked out. It is necessary to know the diameter of the commutator, the number of commutator parts, the width of the brush, and the number of turns per coil; that is to say, the design has to be practically completed before it can be tested as to whether the reactance voltage will be satisfactory. A formula was published by H. A. Mavor, in 1902, which gives substantially the same results, and is in fact practically based on the same reasoning. It is as follows:—

$$\rho = \frac{\text{E C} m \, 20 \times \pi \times l}{\text{P N}}$$

Where ρ is the reactance voltage,

E the electromotive force generated in the armature,

C the current at full load,

l the core length in inches,

P the number of poles,

N the number of magnetic lines per pole,

m the number of turns per section,

substituting

$$\rho = \frac{500 \times 200 \times 1 \times 20 \times \pi \times 8}{6 \times 4{,}350{,}000} = 1\cdot 93,$$

a value sufficiently near the 1·7 obtained by the former method, and within the limit of 2 volts which has been agreed on as the limiting value of reactance voltage to be aimed at.

The other considerations affecting the commutation of the machine are, as has already been seen, that the number of commutator parts should not be too small, and that the armature ampere-turns should not be too great. A safe rule is that the armature ampere-turns per pole should not be in excess of the ampere-turns on each magnet coil. In the present case the armature ampere-turns per pole are 4,730 (see page 70), and the magnet winding gives 7,200 ampere-turns; this is well within the mark and, as already stated, the commutator parts per pole should, for a 500-volt machine of this size, be in the neighbourhood of 50.

If the armature ampere-turns are excessive, they can be diminished, if lap winding is used, by increasing the number of poles, and therefore the number of paths through the armature from brush to brush. The examination of the E.M.F. formula will show that the number of bars per pole, and therefore the number of turns per pole, will not be altered, but by increasing, say, from 6 to 8 poles, the number of paths through the armature is increased from 6 to 8, and the current carried by each turn is therefore reduced in the proportion of 6 to 8. The armature ampere-turns per pole will therefore have only three-quarters of their former value. This consideration in the main determines the number of poles a generator shall have. When a wave winding is used, this method of reducing the armature ampere-turns is not available. From other considerations, however, it is not usual to use wave windings on large machines, and it is in the case of these machines only that difficulty arises from too large a number of armature ampere-turns.

Whilst it is highly important that every attention should be given to obtaining good electric and magnetic constants for the dynamo, it should be observed that sparking is frequently due to mechanical defects in the commutator or brush gear. If the commutator surface is not truly cylindrical, or the brush gear is not such as to insure good contact between commutator and brush, the machine will spark, however good the design.

§ 9. **Commutator and Brushes.**—The principal dimensions of the armature and magnets have now been determined, but as yet nothing has been done with the commutator. The brush area required to carry 200 amperes will be about 7 to 8 square inches; a safe rule being to work carbon brushes at a surface density of not more than 25 to 30 amperes to the square inch. This, however, must be taken as only tentative, the real test being that the watts lost on the commutator are not excessive. These watts are made up of two parts, the $C^2\omega$ loss due to the resistance of the brush and the loss due to friction. As this is a six-pole machine, there will be six points of commutation where brushes can be put on the commutator. There can therefore be put on 6 sets of brushes, 3 positive and 3 negative. Assume that 9 square inches are required to carry the current, this will mean that there should be about 3 square inches of carbon brush at each commutating point. Say that there are at each point 2 brushes, each 2″ by $\frac{3}{4}$″. This will give 9 square inches to carry the 200 amperes, which should be amply sufficient. The contact resistance will of course vary with the pressure.

It is found unnecessary to put very high pressure on the brushes—with ordinary quality of carbon a pressure of $1\frac{1}{4}$ lb. per square inch is sufficient. Greater pressure does not materially reduce the contact resistance, whilst it of course increases the friction. With this pressure, namely $1\frac{1}{4}$ lb. per square inch, the resistance may be taken at ·03 ohm per square inch of contact. In order to calculate the friction losses, the coefficient of friction between carbon and copper must be known. This is very ordinarily taken at ·3.

The diameter of the commutator should be kept large enough to give room for the required number of commutator parts, without making these too narrow. A good width for a commutator segment is something in the nature of $\frac{1}{4}''$. On the other hand, the diameter is limited by considerations of the highest allowable peripheral speed; this should not exceed 2,500 to 3,000 ft. per minute; at higher speeds, unless special precautions are taken, good brush contact is difficult to obtain.

The diameter of the commutator is also limited by the necessity of making connections with the armature winding, and it must be kept as much below the diameter of the armature as is necessary to insure room for good connection being made. Copper strips are usually fixed to the copper segment by soldering and riveting, and are then shaped to receive the bars which are soldered into them. In the present case, for instance, where the armature slot is one inch deep, the diameter of the commutator should not exceed $28'' - (1 \times 2)$, that is $26''$, and it would be advisable for ease in making the connection to keep it at least $2''$ below this. As a matter of fact, however, there is no need for so large a diameter, 268 commutator segments are required, and if each be made $\frac{1}{4}''$ wide at the top, this will give a circumference of $67''$ for the outside of the commutator, and therefore a diameter of $\frac{67}{\pi} = 21''$.

The length of the segments must be sufficient to take two $2''$ brushes side by side, allowing for clearance at the ends and between the brushes, say $5''$.

The current has to pass through both positive and negative brushes. The resistance of one of these sets can first be calculated. Thus:—Area of positive brushes 9 square inches. Resistance $= \frac{\cdot 03}{9} = \cdot 0033$ ohm. $c^2 \omega =$

$(200)^2 \times \cdot 0033 = 132$, but the same number of watts are lost at the negative brushes and the total $c^2 \omega$ watts are therefore 264, say 260.

For the friction losses, there are altogether 18 square inches of contact, and the pressure being $1\frac{1}{4}$ lb. per square inch, the total pressure of the brushes will be $22\frac{1}{2}$ lb. Multiply this by ·3, the coefficient of friction, $22 \cdot 5 \times \cdot 3 = 6 \cdot 75$ lb. is obtained as the friction on the commutator. The peripheral speed is $\frac{21 \times \pi \times 420}{12} = 2{,}300$ ft. per minute, and the total work done against friction is therefore $2{,}300 \times 6 \cdot 75 = 15{,}400$ ft.-lb. per minute; convert this into watts, since all the other losses are in watts, $15{,}400 \times \cdot 023 = 350$ watts. (Note :—Foot-pounds per minute divided by 33,000 = horse-power, and one horse-power = 746 watts; therefore to convert foot-pounds per minute to watts, multiply by $\frac{746}{33{,}000} = \cdot 023$.) The total watts lost on the commutator are thus found to be $260 + 350 = 610$, and the cylindrical surface of the commutator is $21 \pi \times 5 = 330$ square inches; the watts lost per square inch are therefore $\frac{610}{330} = 1 \cdot 86$. The losses may safely amount to from 2 to 2·5 watts per square inch without giving a temperature rise on the commutator of more than 70° F., and it may therefore be considered that the commutator is quite safe as regards temperature rise.

The behaviour of carbon brushes on a copper commutator is, however, not very well known, and actual results in commutator heating differ from the calculated perhaps more frequently and more materially than in any other point of design. An interesting series of experiments on this subject was described in a paper read by Professor

Baily before the Glasgow section of the Institution of Electrical Engineers.*

§ 10. **Efficiency.**—This gives the complete dimensions, electrical and magnetical, of the machine. Each part has been tested from the point of view of heating as the calculations proceeded. The question of commutation has also been considered, and there remains to test the design for efficiency. The efficiency of the machine is defined as the ratio of watts output to the watts output plus the losses. In order to calculate it, the total losses must therefore be found; most of these have already been calculated in dealing with the question of heating.

The following table gives all the losses on the machine:—

C^2R in the armature . .	2,460	2,720
C^2R in the commutator . .	260	
C^2R in shunt winding . .	1,260	4,980
Armature iron	1,870	
Commutator friction . . .	350	
Friction and windage . .	1,500	

The last loss is extremely difficult to predetermine, as it varies largely with the conditions of the bearings, and also with the method of ventilating the armature. It may be taken in a machine of this size as being approximately $1\frac{1}{2}$% of the total output, that is 1,500 watts.

Of the losses given above, the first two are evidently dependent on the load, they vary as the square of the current. The other four are practically independent of the load, and may be called the fixed losses. The efficiency at full load will be obtained by adding together all these losses and dividing the total watts output of the machine by the watts output plus the losses. In the same way the efficiency at $\frac{1}{2}$ or at any other load may be found, the

* See *Journal* of the Institution of Electrical Engineers, No. 181, Vol. XXXVIII.

constant losses remaining the same, but the variable losses being altered in the ratio of the square of the current. Thus the efficiency is found to be—

	FULL LOAD (100 KILOWATTS)	$\frac{3}{4}$-LOAD (75 KILOWATTS)	$\frac{1}{2}$-LOAD (50 KILOWATTS)
Variable losses	2,720	$2{,}720 \times \frac{9}{16} = 1{,}520$	$2{,}720 \times \frac{1}{4} = 680$
Fixed losses	5,000	5,000	5,000
Total losses	7,720	6,520	5,680
Efficiency	$\frac{100{,}000}{107{,}720} = {\cdot}925$	$\frac{75{,}000}{81{,}520} = {\cdot}915$	$\frac{50{,}000}{55{,}680} = {\cdot}9$
	92½ per cent.	91½ per cent.	90 per cent.

These efficiencies are quite reasonable efficiencies to expect from such a machine, and the design may thus be accepted as satisfactory in this respect, as also from the point of view of heating and of commutation.

CHAPTER IV

CONTINUOUS-CURRENT MOTOR

§ 1. Open and Enclosed Type Motors. — A continuous-current motor may be of exactly the same construction as a continuous-current generator. In fact, the same machine may be indifferently used as a generator or as a motor. If the terminals of the machine are connected to supply mains it will take electrical energy from this source of supply and convert it into mechanical energy.

Motors, however, are frequently made in what is known as the enclosed or protected type. The ring forming the magnet yoke is extended to produce a steel shell which will enclose and protect all the working parts of the machine from mechanical damage. At each end of the shell thus formed is fixed an end-plate or frame which carries the bearings for the armature shaft. The end-plates and also the magnet shell, at the commutator end, are provided with large openings, sometimes left open, sometimes protected by a metal grid or filled in with metal gauze, so that at the same time that the working parts of the machine are thoroughly protected from mechanical damage, there is nevertheless free access of air for cooling purposes.

Occasionally motors have to be totally enclosed, that is, all openings in the end-plates and in the shell must be filled in with solid metal. This is a necessity in cases where the motor is to work in a wet place where water would gain access to electrical parts and damage the insulation. The total enclosing of a motor, however, is very unadvisable, and should never be resorted to if it can possibly be avoided. The want of ventilation very largely increases the heating so that, even with the increased temperature rise which is usually allowed on totally enclosed motors, the output of a machine of given size has to be very greatly reduced when the machine is made totally enclosed instead of being provided with ventilating grids.

Whilst both open and enclosed types of machine may be used either as generator or motor, it is nevertheless usual to find all generators made of the open type. Motors are found of both types, but in these the enclosed type probably prevails. The reason for this preference is doubtless that whilst the generator is usually placed in an engine-room, and has the benefit of the constant attention of an attendant, motors are scattered all over the workshops in all sorts of situations. They are there exposed to dirt and dust, and are often placed in such inaccessible positions that they get very little attention. If they had no protection they would, when placed amongst the tools of an engineering works, for instance, be very liable to mechanical damage, and for these reasons they are generally afforded the protection of an enclosed shell.

This, however, does not apply to motors of the larger sizes, say about 100 H.P., where the construction becomes mechanically difficult, and where the open type is therefore preferred. Motors, like generators, may be either series, or shunt, or compound wound. For ordinary industrial purposes the shunt-wound motor is the one most frequently used. The excitation of the machine is independent of the load since the magnet winding is connected directly across the mains, and the speed will, therefore, be approximately constant. This is the condition which is most convenient for driving shafting, and for general shop purposes.

§ 2. **Principle of Back E.M.F.**—The principle on which a continuous-current machine works as a motor is as follows. When a current is passed through the armature winding, the armature conductors, which are at the moment in a strong magnetic field, will tend to move in a direction at right angles to this field. This will give a torque tending to rotate the armature. But as soon as rotation begins, the armature conductors are cutting lines of magnetic force, and, therefore, an E.M.F. is produced in the winding. This E.M.F. is in such a direction as to oppose the flow of the current in the armature bars, and is on that account generally referred to as the back E.M.F. As the speed of the armature increases, this back E.M.F. is also increased, and cuts down the value of the current until the torque due to the current is only just sufficient to overcome the resistance of the load; thus the current automatically adjusts itself to the load put on the motor.

If more load is put on the machine, it will slow down slightly, the back E.M.F. will be smaller and more current will flow through the armature. If C represents the current in a motor armature, E the E.M.F. at the mains, ω the resistance of the armature, and e the back E.M.F. in the armature conductors, then

$$C = \frac{E - e}{\omega} \qquad \therefore C\,\omega = E - e,$$

multiply both sides by C

$$\therefore C^2\,\omega = E\,C - e\,C,$$

$$\text{or } e\,C = E\,C - C^2\,\omega$$

but E C represents the watts supplied from the main circuit, and $C^2\,\omega$ the watts lost in the armature copper, the term $e\,C$ therefore represents the watts available for transformation into mechanical energy. From this it is seen that the amount of electrical energy capable of conversion into mechanical is equal to the current multiplied by the back E.M.F. It must, however, be noted that the whole of this is not available as useful work; the losses occurring in the motor itself must be deducted. The efficiency of the motor can therefore never exceed $\frac{e\,\text{C}}{\text{E}\,\text{C}} = \frac{e}{\text{E}}$ and in practice will always be less than this amount. And $e\,\text{C}$ is less than $\text{E}\,\text{C}$ by the amount $\text{C}^2\,\omega$. In order, then, that a reasonable efficiency should be obtained, the value of ω must be kept small. This is also necessitated by the fact that if the watts lost in the armature winding $\text{C}^2\,\omega$ be large, the armature will get too hot.

At the start, when the motor is standing still, $e = 0$ and therefore $\text{C} = \frac{\text{E}}{\omega}$. Since ω is small, the current at starting will be very large, many times the normal value of the current. Hence the necessity of using a starting resistance, which is inserted in series with the armature circuit, and gradually cut out as the motor accelerates. This resistance is chosen so as to keep the current down to some predetermined value, and thus avoid possible damage to the motor, generating plant, and switch gear, from the passage of an excessive current, and also possible mechanical damage to the motor and driven machinery from too rapid and sudden an acceleration.

§ 3. Normal Rating of a Motor.—It follows from the above considerations that a motor may take from the mains a much greater current than that at which it is rated, and therefore give a much greater horse-power than that marked on its name-plate. A motor, for instance, rated at 10 H.P. will, if called upon, do 20, 30, or even more horse-power, but unless the design has been quite unduly liberal, it will get extremely hot if required to carry the larger current for any length of time, and will spark violently at the brushes.

The motor is rated at 10 H.P. because it will carry the current corresponding to 10 H.P. with the specified temperature rise and efficiency, and without sparking at the commutator, not because it is incapable of doing more than 10 H.P. if called upon to do so.

On referring to Chapter III. it will be seen that the rating of a motor depends on exactly the same considerations as that of a generator.

§ 4. Design of 10 H.P. Motor at 600 Revolutions.— As an example, let it be required to design a motor to give 10 B.H.P. when running at 600 revolutions per minute on a 500-volt circuit. Many of the calculations required will be identical with those for the continuous-current generator, and will, therefore, serve as an additional arithmetical example of the principles involved. The first question to be solved is to determine the current which the motor will be required to take at full load. In order to do this, it is necessary that the efficiency of the motor should be known, but as this cannot be calculated until the design is complete, an efficiency is in the first place assumed, subject to a revision of the calculations, should this be found necessary when the real efficiency of the motor is ascertained.

For a motor of the required output it is safe to assume an efficiency of about 83%, which means that in order to obtain 10 B.H.P. from the motor, it will be necessary to put in $\frac{10}{\cdot 83} = 12$ E.H.P., but one horse-power is equal to 746 watts, and therefore the necessary input will be $12 \times 746 = 9{,}000$ watts. The watts are the product of the volts and amperes, and, therefore, the current at full load must be $\frac{9{,}000}{500} = 18$ amperes.

The diameter and length of core of the armature are the most important dimensions of the machine. A tentative value for these may be found, as in the case of the generator, by using a curve connecting d^2l and D, which is given in Fig. 39. This curve differs from that on page 50, because it is drawn for enclosed-type machines, not for open-type. In this instance the input to the machine

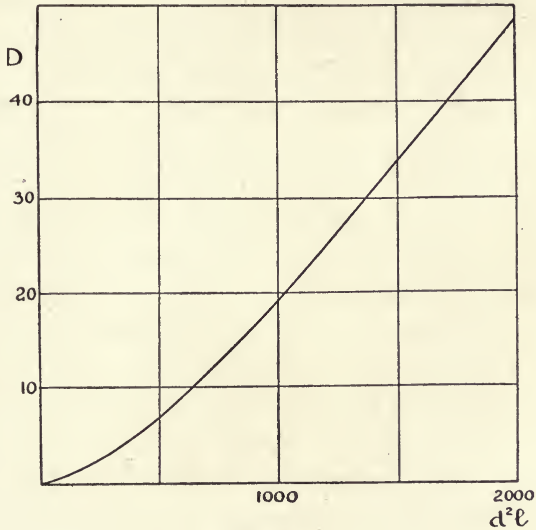


FIG. 39.

is 9,000 watts and the speed is 600 revolutions per minute. The value of D is therefore $\frac{9,000}{600} = 15$. The corresponding value of d^2l from this curve is about 864. Values of 12″ for the diameter and 6″ for the length of core will satisfy this. The same considerations already pointed out in Chapter III. determine the choice of relative length and diameter, and the same considerations which apply to the generator also determine the choice of the number of poles in a motor.

According to present practice this machine would have 4 poles. The number of bars required to give the necessary back E.M.F. can now be calculated by using the formula $Z = \frac{E \times 10^8 \times 60}{P N R}$ (p. 54). The back E.M.F., E, is equal to the E.M.F. of supply less the volts dropped in the armature circuit. Assuming these to be 23 volts, that is, rather more than 4%, the back E.M.F. will be 477 volts. The speed is known to be 600 revolutions, and the number of poles is 4, and it is only required to find the value of N, the number of magnetic lines per pole, in order to solve the equation giving the number of bars. Since the machine has 4 poles the pole pitch is equal to $\frac{12\,\pi}{4} = 9{\cdot}5''$, and if the pole arc be taken as ·7 of the pole pitch, its value will be $9{\cdot}5 \times {\cdot}7 = 6{\cdot}6''$. The area of the pole shoe will be the pole arc multiplied by the length measured parallel to the shaft; this length is usually taken the same as that of the armature core, in this instance 6″.

The area of the pole shoe will thus be $6{\cdot}6 \times 6 = 39{\cdot}6$ square inches, and this multiplied by 6·45 gives 260 square centimeters. The value of 7,000 lines per square centimeter in the air gap is a fairly usual value to work at, and will give a value for N, the number of lines per pole, of $260 \times 7{,}000 = 1{,}800{,}000$ lines per pole. These values can now be substituted in the E.M.F. formula

$z = \frac{477 \times 10^8 \times 60}{4 \times 1{,}800{,}000 \times 600} = 660$ bars to be connected in series. If a wave winding is used, the total number of bars will then be 1,320, but if lap winding is used, double this number of bars, 2,640, will be required.

The wave winding would in this case be chosen in preference to lap, from the point of view of convenience of winding. Such a number as 2,640 wires would be inconveniently large to wind on the armature, and the individual wires each having to carry only $4\frac{1}{2}$ amperes, one-quarter of the full current, since a lap winding gives four paths in parallel through the armature, would be of small section, and consequently a very large proportion of the space in the slots would be taken up by insulation. It is, therefore, better to choose the smaller number of bars, 1,320, and make use of a wave winding.

It is then necessary to put on the armature 1,320 bars, and each of these bars will have to carry half of the full load of current, that is 9 amperes. The ultimate criterion of the bars being sufficiently large is that the number of watts lost in the copper winding shall not be too high, but as this loss can only be calculated when the winding has been determined upon, it is best first to settle the size of bar from consideration of the current density at which it is advisable to work. The density of 2,500 amperes to the square inch, going up to as high as 3,000, is usual in this size of machine. A No. 16 S.W.G. copper wire has an area of ·0032 square inch, and will give a current density of $\frac{9}{\cdot 0032} = 2{,}800$.

§ 5. Number of Turns Allowable per Commutator Section and Reactance Voltage.—In Chapter III., in the case of the generator where the number of bars was comparatively small, it was advisable, after the number of bars and their section had been determined, immediately to proceed to see how they could be disposed on the armature and to make sure that there was room to accommodate the requisite number. With a winding of a comparatively large number of small wires, it is better first to consider how many turns may be connected between two commutator parts.

In large machines, carrying a considerable current in each bar, it is seldom possible to have more than one turn per commutator part without unduly increasing the reactance voltage of the short-circuited coil. In a small machine, where each wire carries only a small current, the case is different, and it is, therefore, advisable to calculate the maximum number of turns per commutator part allowable before proceeding to consider how the wires can best be allocated in the slots.

It should be clearly observed that the difference in the order of operations here suggested is due not to the fact that the machine now considered is a motor, whilst that in Chapter III. was a generator, but to the fact that the specified outputs lead in Chapter III. to a bar winding, and in this case to a wire-wound armature.

Using the formula $\rho = \frac{\text{E C} m \, 20 \times \pi l}{\text{P N}}$ (see page 81), which gives the reactance voltage, all the quantities are known except m, the number of turns per section; the reactance voltage should not exceed 2 volts, if good commutation is to be obtained without change of brush position.

Substituting $2 = \frac{500 \times 18 \times m \times 20 \times \pi \times 6}{4 \times 1 \cdot 8 \times 10^6}$

from which $m = \frac{2 \times 4 \times 1 \cdot 8 \times 10^6}{500 \times 18 \times 20 \times \pi \times 6} = 4 \cdot 3$,

that is 5 turns per section would give a reactance voltage greater than 2, and thus 4 turns is the largest number that should be used.

Giving to m the value 4, and again substituting in the formula for reactance voltage it is found that

$$\rho = \frac{500 \times 18 \times 4 \times 20 \times \pi \times 6}{4 \times 1 \cdot 8 \times 10^6} = 1 \cdot 88 \text{ volt.}$$

§ 6. **Number and Size of Slots.**—The number of coils in each slot may be three, four or five; a greater number is not advisable as it may cause the commutator to mark through the coils not being in exactly the same magnetic field as the moment of reversal. It should be pointed out that the freedom of choice in this respect is more limited with a wire winding than with a bar winding. The bar, so long as its section is kept the same, may be made of such depth and width as are found most suitable to give convenient slot dimensions, the wire is necessarily round, and the number of wires put in one slot must be chosen so as to give suitable slot dimensions; no adjustment can be made by varying the relative depth and width of the conductor such as is possible with a rectangular bar.

In the present case it will be found that if more than three wires are placed side by side in the slot, the slot width will become inconveniently large compared to its depth. Let it then be settled to put three coils in each slot, and that each coil is to have four turns; this means that $3 \times 4 \times 2 = 24$ wires will be required in each slot (there are two wires to one turn), and since a total of 1,320 wires is required, the number of slots will be $\frac{1,320}{24} = 55$.

The insulation of the wire will be double cotton covering, and to allow for this ·012″ must be added to the diameter of the wire. The wires in one slot will be divided into 12 wires forming the top halves of coils and 12 forming the bottom halves, and each of these bundles of 12 wires will be taped together with cotton tape as a protection from contact with the iron of the armature. The allowance for this tape is ·035″, added to the thickness and to the depth of each bundle of wires (Fig. 40).

In addition to the above insulation on the wire, an insulating trough of presspahn, leatheroid, or some such material is put in the slot before the coils are put in place; for machines intended to work in high temperatures, and for those working in a damp atmosphere, this slot insulation should consist of mica. Mica must also be used for machines working at high potentials, but for motors intended to work on circuits not exceeding 500 volts' pressure, and under normal conditions, paper or one of the above-mentioned materials answers every purpose.

In any case, an allowance has to be made for the thickness of the lining, ·03″ on each side is sufficient for this, if the pressure does not exceed 500 volts.

The slot will have to accommodate eight No. 16 wires in the depth, and three such wires side by side. It is always advisable when possible to have an arrangement of the wires giving the necessary number for one coil (in this case 8) in one vertical row, as this gives the most convenient method for bringing out the ends of the coil to the commutator. Some different arrangement not fulfilling this condition is sometimes almost forced on the designer, but it usually leads to the ends of the coils coming out in inconvenient positions in the slot, and should be avoided whenever possible.

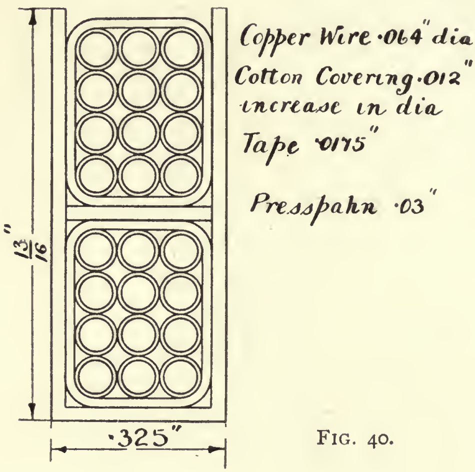


FIG. 40.

From copper wire tables (see Appendix), it is found that the diameter of bare No. 16 S.W.G. copper wire is ·064″, and the slot dimensions are, therefore,

	DEPTH	WIDTH
D.C.C. copper wire	(·064 + ·012)8 = ·608″	(·064 + ·012)3 = ·228″
Tape	·07	·035
Slot insulation	·055	·055
Slack	·070	·007
	·803, say $\frac{13}{16}$″	·325″

§ 7. **Inductions in Teeth.**—To make sure that this is a suitable arrangement, and that there is room for this number of slots, it is well now to calculate the resulting densities at the bottom of the teeth as follows. The tooth pitch at the armature surface is $\frac{12\,\pi}{55} = \cdot 678$, $12\,\pi$ being the circumference of the armature. From this, subtract ·325, the width of the slot, and there remains for the width of each tooth at the top ·353″. Similarly at the bottom of the slot, the diameter is $12'' - 2 \times \frac{13}{16} = 10\frac{3}{8}''$, and the tooth pitch is $\frac{10\cdot 375\,\pi}{55} = \cdot 593$. Again subtracting ·325″, the width of tooth at its narrowest part is ·268″. And since the pole arc is ·7 of the pole pitch, the average number of teeth under one pole will be $\frac{55}{4} \times \cdot 7 = 9\cdot 6$. To make allowance for the fringing of the magnetic field, it will be necessary to add 10% to this, and the number of teeth carrying the magnetic lines from one pole becomes 10·5.

It is now necessary to determine the effective length of iron in the armature core. The total length of core is 6″, and it is proposed that there should be two ventilating ducts, each ½″ wide (Fig. 41); deducting these, there remains 5″, of which 10% will be insulation between the discs and the remaining 90% iron. The effective length will therefore be $5 \times \cdot 9 = 4\cdot 5''$. The amount of iron at the bottom of the teeth carrying the flux from one pole will, therefore, be $10\cdot 5 \times \cdot 268 \times 4\cdot 5 = 12\cdot 4$; multiply this by 6·45 to bring it to square centimeters, $12\cdot 4 \times 6\cdot 45 = 81\cdot 5$ square centimeters, and the value of B, the induction at the bottom of the teeth, is therefore $\frac{1{,}800{,}000}{81\cdot 5} = 22{,}000$. Calculated in exactly the same way, the value of the induction at the top of the teeth is found to be

$$\frac{1,800,000}{110} = 16,400.$$

These inductions are quite suitable values to work with, and it may therefore be concluded that the proposed number of wires and their arrangement in the slots can be carried out on an armature core of the given dimensions.

§ 8. **Armature Losses.**—The losses both in the copper winding and also in the iron core may now be calculated.

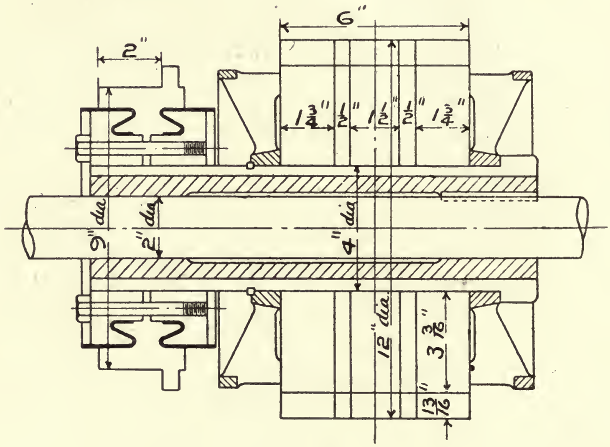


FIG. 41.

The length of each coil is calculated on the principles already used in the case of the generator. In the present case one end connection is about $5\frac{3}{4}''$ long (Fig. 42), and the part of the bar lying in the slot will be $6''$. The total length of the bar is therefore $6 + 5\frac{3}{4} \times 2 + 1\frac{1}{2}'' = 19''$, and there are 1,320 bars, the total length of winding is therefore $\frac{19''}{12} \times 1,320 = 2,100$ ft. From the table in Appendix it can be found that the resistance of 2,100 ft. of No. 16 S.W.G. copper wire = 6·4 ohms. Since this is disposed on the armature in two paths connected in parallel, the resistance from brush to brush will be $\frac{6 \cdot 4}{4} = 1 \cdot 6$ ohm, and the total current is 18 amperes, the volts dropped in the armature winding will, therefore, be $1 \cdot 6 \times 18 = 28 \cdot 8$, and the watts lost are the product of the volts by the amperes, and are equal to $28 \cdot 8 \times 18 = 510$. Of these watts, the part lost actually in the slots is proportional to the ratio of the length of winding lying in the slot to the total length of winding, that is the watts lost in the slot =

$$510 \times \frac{6}{19} = 162.$$

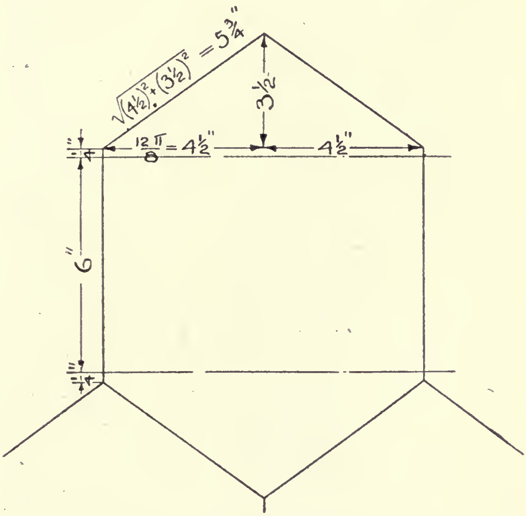


FIG. 42.

The depth of core below the teeth should be made such as to give a magnetic density of about 10,000. But the total number of lines from each pole is 1,800,000, and dividing this by 10,000 gives 180 square centimeters as the necessary area, 180 square centimeters = 28 square inches, and as the lines divide into two paths, each of these two paths must have a section of 14 square inches. It has already been found that the effective length of iron parallel to the shaft is 4·5″, the depth of the discs below the slot must, therefore, be $\frac{14}{4 \cdot 5} = 3 \cdot 1''$, say $3\frac{3}{16}$, and from this the inside diameter of the core discs can be determined. The external diameter being 12″, subtract from it twice the tooth depth and twice the depth of core. The inside diameter $= 12 - 2(\frac{13}{16} + 3\frac{3}{16}) = 4''$ (Fig. 43).

The weight of the core and of the teeth can now be calculated. The weight of core $= \frac{\pi}{4}(12^2 - 4^2) \times 4{\cdot}5 \times {\cdot}28 = 126$ lb., less the weight of iron in 55 slots each ${\cdot}325'' \times \frac{13}{16}''$; this is equal to $55 \times {\cdot}325 \times \frac{13}{16} \times 4{\cdot}5 \times {\cdot}28 = 18{\cdot}2$ lb. The total weight of iron in core and teeth is therefore $126 - 18{\cdot}2 =$ say 108 lb.

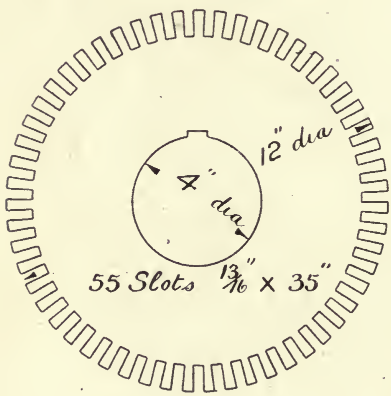


FIG. 43.

Also the magnetic density B in the core is 9,800, and the periodicity or number of reversals of the magnetism per second is $\frac{600}{60} \times 2 = 20$, $\frac{600}{60}$ being the number of revolutions per second, and 2 the number of pairs of poles. $B \times \sim$ is therefore equal to 196,000, and from the curve given on page 61 it is found that the corresponding loss is 2·05 watts per lb.

The total watts lost in the iron are thus found to be $108 \times {\cdot}205 = 222$. Add to this 162 watts lost in that part of the copper winding which lies in the slots, and the total loss on the core will be 384. If taken over the whole armature, including end windings, the copper watts lost are 510, and adding the iron losses, the total losses over all are 732. Again, the cooling surface calculated as the surface of the core only is $12\,\pi \times 6 = 226$, and the armature length over the windings will be about 13″ (Fig. 42), the total armature cooling surface thus being $12\,\pi \times 13$

$= 489$. The watts lost per square inch are therefore $\frac{384}{226} = 1{\cdot}7$ on the core only, and $\frac{732}{489} = 1{\cdot}48$ over all. The peripheral speed of the armature in feet per minute is $\frac{12\,\pi}{12} \times 600 = 1{,}890$. In Fig. 44 are given two curves drawn in exactly the same way as the curve in Fig. 33,

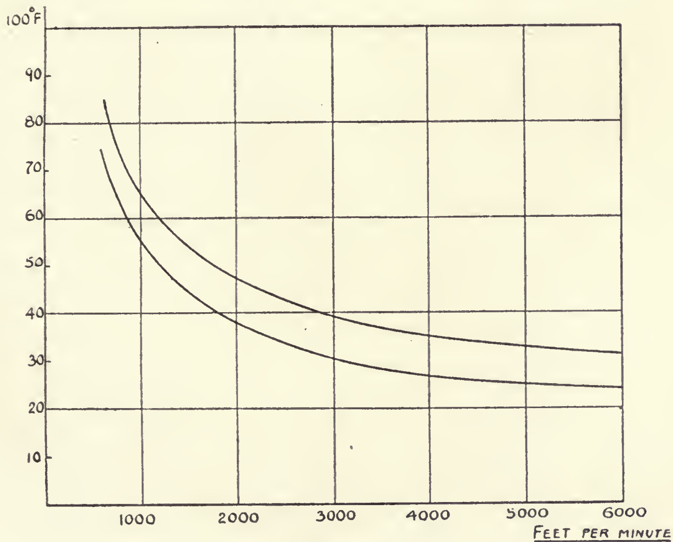


FIG. 44.

but applying in this case to enclosed-type machines. The lower curve gives the rise in degrees Fahrenheit to be expected from the loss of one watt per square inch calculated on the core surface only; and the upper curve the rise per watt per square inch calculated on the over-all dimensions of the armature. The two values of temperature rise obtained should be approximately equal if the iron and copper losses have been properly proportioned. From Fig. 44 it is found that one watt per square inch calculated on the core only will give a rise of 38° F., and 38 ×

$1{\cdot}7 = 65°$ F. Calculated over all, one watt per square inch gives a rise of 48° F., and $48 \times 1{\cdot}48 = 71°$ F. The temperature rise on the armature may therefore be expected to be about 70° F., and the design considered satisfactory as far as the armature temperature rise is concerned.

The dimensions of the steel magnets may now be considered.

§ 9. Dimensions of the Magnets.—The magnets will have to be of sufficient section to carry the full number of magnetic lines, that is 1,800,000, and in addition to this whatever number of lines is allowed for leakage. If the shell is made of cast steel, as is often the case, the total magnetic density should not exceed about 13,000 lines to the square centimeter, which gives a required area of $\frac{1,800,000}{13,000} = 138$ square centimeters, multiply by 1·2 to allow for leakage, and the area must be 166 square centimeters $\frac{166}{6{\cdot}45} = 26$ square inches, and supposing that the magnet core, measured parallel to the shaft, is made the same length as the armature, namely 6″, this will give a width of core, measured perpendicularly to the shaft, of $\frac{26}{6} = 4{\cdot}3''$, say $4\frac{1}{4}''$.

To determine what portion of the yoke is effective for carrying lines is more difficult in the case of enclosed machines than in one of the open type. The casting forming the yoke is extended in order to give mechanical protection to the electrical parts. The path of any magnetic lines running through the ends of the casting is evidently much longer than that of lines keeping to the central portion, the magnetic density is therefore reduced in those parts, and they are magnetically less effective. It is a matter of some difficulty to say what allowances must be made for this reduced carrying capacity of those parts of the shell which are far removed from the central band, and the question is still further complicated by the openings which are left in the commutator end for observation and ventilating purposes.

One plan which has been frequently adopted is to make the thickness of the shell only sufficient for mechanical strength, and to increase it over the magnet cores by a band which is made of sufficient thickness to carry the whole of the magnetic flux. In the present instance if this band were made 6″ wide its thickness would have to be about $2\frac{1}{8}''$. Note must be taken of the fact, already pointed out in the previous chapter, that the magnetic lines from any one pole divide into two paths in the yoke, one to the right and one to the left, and that the yoke at any one section need only be half the section of the magnet core.

There is some advantage in laminating the pole shoe of a motor, that is making it of iron plates instead of solid metal. One method of doing this is to stamp the pole shoe and the magnet core out of sheet iron, and build up a sufficient number of plates to give the full width of the core. The plates are riveted together, and the solid block thus formed is drilled and tapped for two bolts which fix it in position in the shell. (See Fig. 45.) When laminated magnet cores are used the shell is often made of cast steel, but not of cast iron.

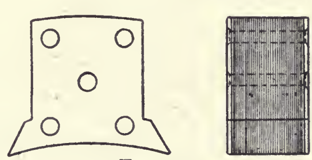


FIG. 45.

The practice of using cast iron for any part of the magnetic circuit does not appear to have any great advantages. The permeability of cast iron is so much less than that of cast steel that a much greater section must be used. If the total flux per pole is kept large, a practice which has many advantages, the weight of the machine becomes excessive. When cast iron is used, the cost, considering the increased section necessary to carry the given number of lines, will be found to be usually in excess of the cost of cast steel to carry the same flux. If there is any place where cast iron may be used to advantage it is certainly in the shell of an enclosed motor, a considerable portion of which is used merely for mechanical protection. Since, however, a shell is usually made of one casting, the whole must be of cast steel or of cast iron, and the extra cost of using cast steel for these parts which are merely mechanical, and do not carry magnetic lines, may be more than compensated for by the saving in weight in those portions which are used magnetically.

The shape given to the magnet core is usually different in enclosed and open-type machines. In the latter, the section of the magnet core is frequently circular, as this shape gives the shortest periphery for a given amount of material, and therefore gives the shortest possible mean

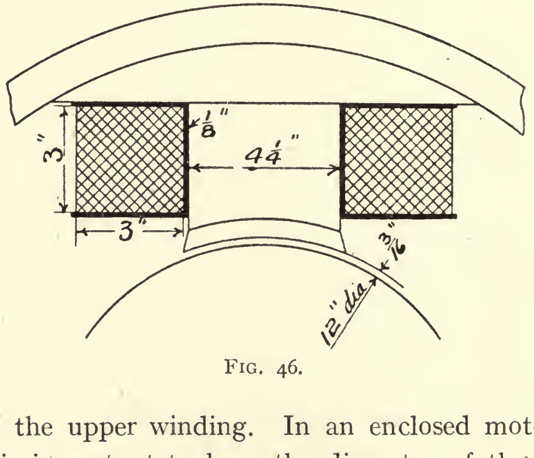


FIG. 46.

turn for the upper winding. In an enclosed motor, however, it is important to keep the diameter of the shell as small as possible. A great deal of material is required in the shell for mechanical protection only, and the weight would therefore become excessive if the diameter were made as large as would be allowable in an open-type machine of the same size. With this end in view the magnet cores are usually made of rectangular section. Fig. 46 shows the machine as actually proposed with magnet cores $4\frac{1}{4}'' \times 6''$ and the magnet coils in place; it is readily seen that if made with circular cores, which to have an equal area must be made $5\frac{3}{4}''$ in diameter, the diameter of the yoke and, therefore, its weight must be increased to make room for the larger coils.

Another modification on the magnet construction which is sometimes adopted is to have no pole shoe fitted, but to make the magnet core of uniform section from the yoke to the air gap (Fig. 47 shows magnet with pole shoe,

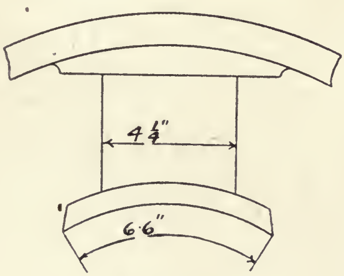


FIG. 47.

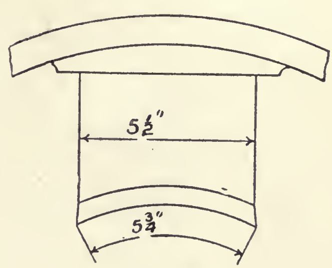


FIG. 48.

Fig. 48 without pole shoe). It is evident that in order to do this, either the pole arc must be considerably shortened or the magnet core must be made considerably wider than is required for the purpose of carrying the magnetic flux. The latter alternative involves not only increased cost in the steel required, but also more copper in the magnet winding since the mean turn is increased.

If on the other hand the pole arc is shortened the area of the air gap carrying magnetic lines is decreased, which involves more copper on the armature, and either a smaller flux must be used or increased densities in the air gap and in the armature teeth. Increased densities mean a greater number of ampere-turns, and therefore more copper in the magnet winding. The plan usually adopted is to in-

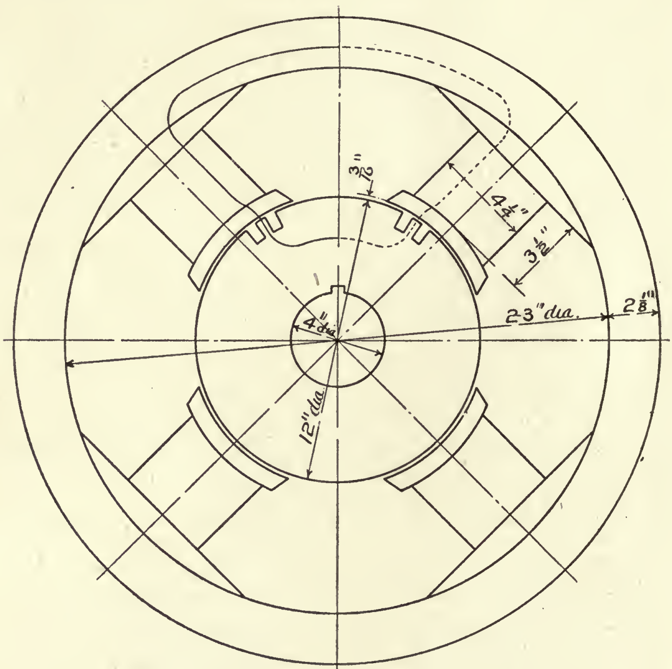


Fig. 49.

crease the magnet width to some extent, and also to shorten the pole arc. It is evident that doing without pole shoes is always more expensive in material, and it is questionable whether the saving in the cost of making and fitting the pole shoes is sufficient to justify this method being adopted.

Instead of fitting the pole shoes on to magnet cores forming part of the shell casting, the magnet core and pole shoe may be cast in one piece, the magnet coil slipped on, and the whole then fitted to the yoke, being held in place by bolts passing through the shell. The cost of this method is about the same as that of fitting on separate pole shoes.

§ 10. **Calculation of Ampere-turns required.**—In the following calculations the magnet is supposed to consist of a cast steel shell, the magnet cores being part of the yoke casting and cast iron pole pieces being fitted. The yoke has a thickened band of metal 6″ × 2⅛″ over the magnet core. Fig. 49 is a section of the machine on which are shown the approximate mean paths of the magnetic lines. From this diagram the length of magnetic path in the different parts of the circuit can be measured, the necessary areas have already been ascertained, and the following table can be constructed, showing the ampere-turns required to give 1,800,000 magnetic lines per pole.

	AREA IN SQ. CMS.	B	LENGTH IN CMS.	H	H × l	A.-T.S.
Air gap .	260	7,000	·475	7,000	3,300	2,660
Armature core	184	9,800	10	1·5	15	12
Teeth top .	81·5	22,000 }	2·06	150	310	244
Teeth roots	110	16,600 }				
Magnet core and yoke	(4¼″ × 6″) 164	11,000 × 1·2 = 13,200	32	18	610	484
					4,235	3,400

The values in this table are obtained as explained in Chapter III., § 6. A slightly different method, however, has been used in dealing with the air gap. The fringe occurring at the edge of the pole shoes has the result of increasing the effective area of the air gap. The fact that the lines "bunch" into the tops of the teeth instead of passing uniformly from magnet to armature results in decreasing the effective area of the air gap; in small machines it is often assumed that these two effects will neutralise one another. Instead, therefore, of making a separate allowance for each, as was done in Chapter III., no allowance has been made for either, it being assumed that they would be approximately equal and, since they are of opposite effect, would cancel out.

§ **11. Winding of Magnet Coils.**—The winding of the magnets may, as in the case of a generator, be either shunt, or series, or compound. The different windings will, in the case of the motor, affect not the voltage, since the E.M.F. of supply is kept constant, but the speed.

The series-wound motor will run at a high speed under light loads, and the speed will fall as the load increases. In all but the smallest motors, where the efficiency is poor, the speed may become dangerously high, if the load is suddenly thrown off, and it is unusual to use series-wound motors except in cases where the conditions are such as to insure some load being always kept on the machine.

Motors for cranes and for traction purposes are usually geared to the load, and the power required to drive the gear will be sufficient to prevent dangerous racing. In all such cases, the use of series winding is allowable. When the motor, however, drives the load through belting, there is a liability, if the belt break, or come off, that the motor should remain connected to the mains with no load on it, in which case the speed may become so high as to damage the armature or commutator, on account of the high peripheral speed.

The winding adopted for motors driving machinery in various industries is usually shunt winding. A shunt-wound motor will run at approximately the same speed, whatever the load, and this condition is the most useful for general purposes. There is, however, a slight drop in speed, as the load increases, due to the increased armature losses. It has been seen in the case of a shunt-wound generator that there is some drop in E.M.F. as the load increases, and this can be compensated for by putting on a few turns of series winding on the magnets in addition to the shunt turns. The same device may be adopted in the case of the motor in order to keep the speed absolutely uniform whatever the load. It should be noted, however, that in the case of the generator, the drop due to the armature resistance is added to the effect of the back armature ampere-turns, and both causes contribute to lowering the E.M.F. with increased load. In the case of the motor, these causes act in opposition. With a given E.M.F. of supply the drop in the armature conductor tends to lower the speed, but the armature back ampere-turns tend to increase it. Fewer series turns would, therefore, be required on a motor than a generator.

It is also of minor importance in ordinary circumstances that a motor should keep at absolutely the same speed, and it is accordingly unusual to find motors compound-wound for the purpose of keeping their speed constant.

Motors are, however, very frequently compound-wound for another purpose. In this case the series turns are wound so as to increase the magnetisation of the machine; they act with the shunt, not in opposition to it. The result of this is still further to increase the change of speed with change of load, but it has also the effect of giving a very strong field at starting, when a big rush of current passes through the series turns. It, therefore, increases the torque at starting, and has the effect of greatly improving the behaviour of the brushes under fluctuating loads, since it causes the field to increase at those times when a big armature current has to be reversed.

A compound-wound motor with the shunt and series coils acting together is therefore a useful machine in cases when the apparatus has to be frequently started under full load or where large and frequent fluctuations of load take place. The exact amount of series winding to be put on for this purpose is indeterminate. The greater the number of series turns the better the starting torque, but this is obtained at the cost of a worse speed regulation. A frequent compromise is to calculate the necessary ampere-turns at full load, and to put on about a quarter of these, as series turns, the other three-quarters as shunt turns.

In the present instance it is proposed to put on the magnets a simple shunt winding. From the table on page 109, 3,400 ampere-turns are required on each pole; add about 10% margin, say 3,700 ampere-turns. It is required to find the proper size of wire, in order to have on each magnet 3,700 ampere-turns, and how many turns of such wire are necessary in order to keep the temperature-rise within the specified limit.

The winding space may be tried as $3'' \times 3''$. Allowing $\frac{1}{4}''$ clearance on each side of the core, $\frac{1}{8}''$ for insulation, and $\frac{1}{8}''$ as clearance, to insure the coil slipping on easily, the mean length of one turn will be

$$2\,(6\tfrac{1}{2} + 4\tfrac{3}{4}) + 3\,\pi + 1'' = 33{\cdot}5''.$$

The cooling surface is taken as one end of the coil added to the surface of the coil round its periphery. The end surface (the area shown in Fig. 50) may be taken as the mean turn $\times\ 3'' = 33{\cdot}5 \times 3 = 100{\cdot}5$ square inches. The outside periphery of the coil is equal to $2\,(6\frac{1}{2} + 4\frac{3}{4}) + 6\pi + 2 = 44''$, and this multiplied by $3''$, the depth of the coil, gives 132 square inches for the sides of the coil. These two areas added together give $132 + 100{\cdot}5 = 232{\cdot}5$ square inches as the value of the cooling surface.

Allowing ·6 of a watt to be radiated per square inch, this allows of 140 watts being lost in each coil. Since the voltage of supply is 500, and there are 4 magnet coils, which will be connected in series, the voltage on each will be $\frac{500}{4} = 125$, then since 140 watts can be lost, the allowable current is $\frac{140}{125} = 1 \cdot 1$ amperes, but the number of ampere-turns required on each coil is 3,700, and the number of turns to be provided must therefore be $\frac{3{,}700}{1 \cdot 1} = 3{,}400$, hence the total length of wire is $\frac{3{,}400}{12} \times 33 \cdot 5 = 9{,}500$ ft.

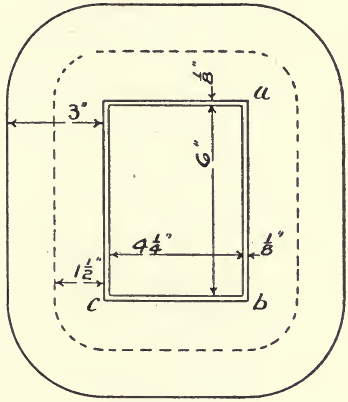


FIG. 50.

Again, if an E.M.F. of 125 volts is to put a current of $1 \cdot 1$ ampere through the coil, the resistance must be $\frac{125}{1 \cdot 1}$ $= 114$ ohms, and the resistance per foot be $\frac{114}{9{,}500} = \cdot 012$ ohm. On reference to copper wire tables, it is found that No. 21 S.W.G. wire has a resistance of ·012 ohm per foot. It is also found that the diameter of the bare wire is ·032″; adding ·012″ to the diameter as an allowance for the cotton covering, the over-all diameter is ·044″. From the 3″ allowed for winding, a space of $\frac{1}{8}$″ on each side should be subtracted for the thickness of tape, which will be used to insulate the coil, leaving $2\frac{3}{4}$″ as actual wire space, and the number of turns that will be got in one layer will thus be $\frac{2 \cdot 75}{\cdot 044} = 62$; 10% must be taken off this, to allow for slack in the winding, leaving 56 as the actual number of turns which will be got in one layer. The total number of turns required has been found to be about 3,400, the number of layers is therefore $\frac{3,400}{56} = 60$. The space required for 60 layers is $60 \times \cdot 044 = 2 \cdot 64''$, add 10% allowance for slack = say $2 \cdot 9''$, so that the magnet coil will easily go into the space, $3'' \times 3''$, previously allowed for it.

The weight of this wire will be $9,500 \times \cdot 0031 = 28$ lb. per coil = 112 lb. for the 4 coils, ·0031 lb. being, as shown in the table, the weight of one foot of No. 21 S.W.G. wire.

§ 12. Dimensions of the Commutator.—The commutator to collect a current of 18 amperes will have to be about 9″ in diameter, and about 2″ long (Fig. 41). Four spindles carrying brushes should be used, and assuming that one brush $\frac{3}{4}'' \times 1\frac{1}{2}''$ is put at each commutating point, there will be altogether 2 positive brushes having a total area of $1\frac{1}{2} \times \frac{3}{4} \times 2 = 2\frac{1}{4}$ square inches. Again taking the contact resistance of carbon as ·03 ohm per square inch, the resistance of the positive brushes will be $\frac{\cdot 03}{2 \cdot 25} = \cdot 0133$ ohm, and the resistance of the positive and negative brushes together will be double this, ·027 ohm. The voltage lost at the commutator will therefore be $18 \times \cdot 027 = \cdot 48$ and $\cdot 48 \times 18 = 9$ watts will be lost.

Again, the peripheral speed of the commutator is $\frac{9\pi}{12} \times 600 = 1,400$ ft. per minute. If the pressure on the brushes be $1\frac{1}{4}$ lb. to the square inch, the total pressure will be $1\frac{1}{4} \times 4\frac{1}{2} = 5 \cdot 6$ lb., and assuming that the coefficient of friction is ·3 this will give a tangential force of $5 \cdot 6 \times \cdot 3 = 1 \cdot 7$ lb., and this multiplied by the peripheral speed gives $1 \cdot 7 \times 1,400 = 2,400$ ft. lb. per minute as the rate of doing work against the friction of the brushes $\frac{2,400}{33,000} = \cdot 073$ H.P., and $\cdot 073 \times 746 = 54$ watts.

There are thus lost altogether on the commutator 9 watts in $c^2 \omega$, and 54 watts in friction losses, giving the total of 63 watts, and the cooling surface from which these are to be got rid of is $= 9\pi \times 2 = 56$, giving $\frac{63}{56} = 1 \cdot 12$ watt per square inch.

It would be quite possible to lose from two to two and a half times this amount of watts without any fear of serious heating, and from this point of view it might be considered that the dimensions of the commutator are too liberal.

It is, however, inadvisable to shorten the commutator to any great extent as the clamping of the commutator segments becomes difficult if they are extremely short. The diameter cannot be reduced, for as there are 165 commutator parts, the thickness of one commutator part and its insulation is already as small as $\frac{9}{165} = \cdot 17''$, and any considerable reduction of diameter would make the parts too thin to be conveniently handled.

Some saving might be effected by reducing the number of brushes to two; it has already been seen (Chapter II.) that with a four-pole wave-wound armature it is possible to collect at two commutating points only; the objections to this course are that there would then be only one positive and one negative brush on the machine; it is always advisable to avoid this when possible, for if one brush only is carrying the whole current it is impossible to lift it off the commutator when the machine is running; any slight accident to the one brush therefore means stopping down, whilst with two or more brushes carrying current it is possible to lift the defective brush off the commutator and keep running with the others.

Also, if the argument in Chapter III., on which is based the calculation of reactance voltage, be referred to, it will be seen that it depends on having only one coil short-circuited between the brushes; but on a four-pole machine having brushes only at two commutating points, there are two such coils in series and the reactance voltage should therefore be doubled.

In practice it is found that there is no such marked difference as indicated above and, as a general rule, a machine which works satisfactorily with four sets of brushes will also give good commutation with two only. There are, however, cases occasionally met with where a machine does not give very good results, and sparks to a slight extent with two sets of brushes, and is very much improved and becomes quite satisfactory with four sets.

§ **13. Calculation of Efficiency.**—To consider now the efficiency of the 10 H.-P. motor dealt with in this chapter, it is merely necessary to collect together the losses which have been already calculated, and see what percentage they bear to the output. It is necessary, however, to make an allowance for the friction at the bearings, and for air friction; this is not readily calculable, but may be taken for such a machine as this, at about $3\frac{1}{2}\%$ of the output, say 250 watts.

The losses then are:—

$C^2 \omega$ in armature winding	510	watts
Iron losses in armature	222	,,
Losses in shunt winding	560	,,
Losses in commutator	63	,,
Friction and windage losses	250	,,
Total	1,605	,,

and the required output is 10 H.P. = 7,460 watts, the input will then be 7,460 + 1,600 = 9,060, and the full load efficiency will be $\frac{7,460}{9,060} = \cdot 83$ or 83%.

§ 14. Comparison of Generator and Effect of Size.—On comparing the methods and calculations in this chapter with those used in Chapter III., it is found that the same considerations determine the chief lines of design whether the machine is to be used as generator or motor.

The output at which the motor may be rated depends, in exactly the same way as that of the generator, on the heating, sparking, and efficiency resulting from a given current. The probable values of these different effects are calculated in the same way, and from the same formulæ, and any difference in the treatment of the example in this chapter from that of Chapter III. is due either to the fact that one machine is of the enclosed and the other of the open type or to the difference in size of the two machines.

The question of keeping the reactance voltage sufficiently low to insure good commutation is one which has to be treated somewhat differently in small and in large machines.

It has already been seen (§ 5 of this chapter) that a 10 H.-P. motor wound for 500 volts may have more than one turn per commutator section; so long as this is the case the commutation of the machine can always be improved by reducing this number.

If, for instance, it had been found in the present example that a value of $m = 4$ in the formula

$\rho = \frac{\text{E}c \times m \times 20 \times \pi \times l}{\text{P N}}$ gave too high a value of ρ

this could have been decreased by making $m = 3$, and correspondingly increasing the number of commutator parts.

As the size of the machine increases, and with it the current carried by each bar, it is soon found that values of m greater than one are not admissible. In such a case, if it is found that the tentative dimensions chosen for any machines give when substituted in the formula too high a value for the reactance voltage, the only course open to the designer is in general to decrease l, the length of armature core, and correspondingly to increase the diameter.

The chief dimensions of the machine as they affect the electrical and magnetic design having now been ascertained,

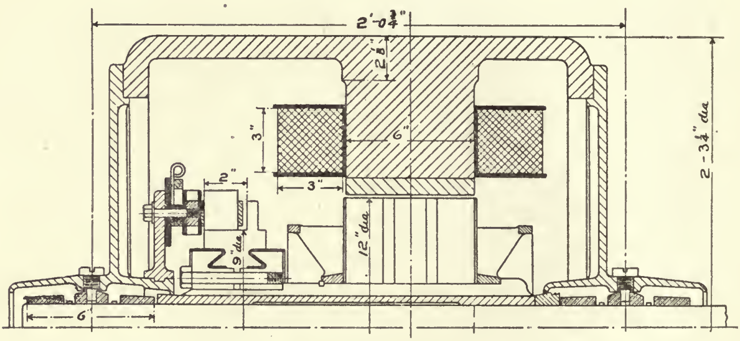


FIG. 51.

the dimensioned sketch (Fig. 51) shows the principal dimensions arrived at, and Fig. 52 shows the external appearance of the machine.

§ 15. Starting Resistance.—Since a starting resistance is usually required for use with a motor it may be well to consider shortly what amount of resistance should be provided for this purpose.

The resistance of the armature from brush to brush in the 10 H.P. motor has been found to be 1·6 ohm. If it were connected directly across a 500-volt circuit, the current momentarily passing through the armature would be $\frac{500}{1 \cdot 6} = 310$ amperes, a very excessive current since the circuit is calculated to carry 18 amperes only; as, however, the armature will immediately accelerate and generate a back E.M.F. which will rapidly cut down the current, it is

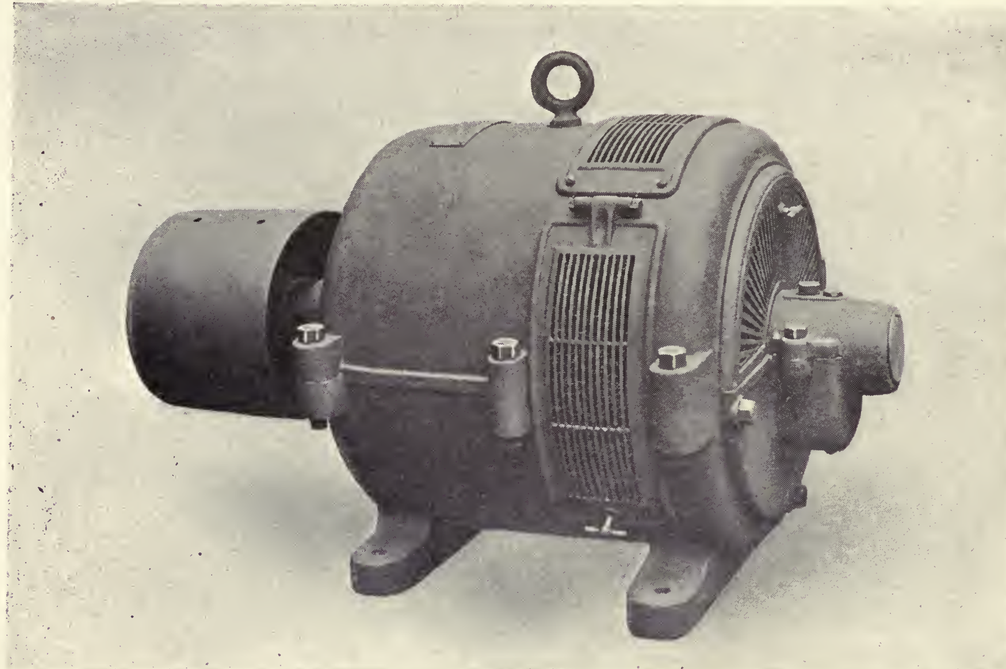


Fig. 52.—Continuous-Current Motor.

quite legitimate to allow more than the full load current of the machine to pass through on the first step of the starting resistance.

The greater the current allowed to pass, the greater the bulk and cost of the resistance, for the size of resistance required varies with the watts it has to dissipate, and these vary directly as the resistance, but as the square of the current. The suitable resistance to use in any case depends upon the conditions under which the motor is required to start. If it starts light, a resistance allowing only half the normal current on the first step may be suitable; if on the other hand the motor starts against a heavy torque, it may be necessary to allow as much as twice the normal current to flow before the motor will start.

A resistance suitable for starting the motor considered in this chapter light, might have about 54 ohms, this added to the armature resistance of 1·6 ohm will give a total resistance in circuit of 55·6 ohms, and the current will thus be $\frac{500}{55 \cdot 6} = 9$. The size of wire used should be such as to carry this current for, say, half a minute, without dangerous heating.

CHAPTER V

MECHANICAL DETAILS

§ 1. Shaft.—The mechanical design of dynamo electric machinery is carried out by means of the same rules as that of any other class of machines. There are, however, some points which are special to dynamo machinery, which will now be considered.

The shaft is calculated of such a diameter as to resist bending. The distance between the bearings is in many machines considerable, and the weight of armature and of commutator supported between them is such as to make the torsional stress negligible in comparison to the bending. In addition, if the armature gets slightly out of centre, there is a strong magnetic pull tending to increase the deflection, and the shaft must be strong enough to resist this also. It is therefore usual to calculate the diameter of the shaft so as to be stiff enough to give a deflection not greater than a certain percentage of the air gap. In present practice there is almost invariably provided a spider which is keyed on to the shaft, and on which the armature discs are threaded. The presence of the spider adds materially to the stiffness of the shaft. It is usual, however, to leave it out of account in the calculations, and to take the benefit of it as an additional factor of safety, and as a set off against any magnetic pull which may be present.

Taking the case of the motor in Chapter IV., the weight of armature and commutator for a 12″ × 6″ machine will be about 260 lb., the distance between the centres of the bearings will be 25″, and the deflection due to this weight for a shaft 2″ diameter would amount to about ·004″.

This is calculated as follows:—

The deflection of the shaft is treated as that of a beam supported at both ends, and having the total weight concentrated at the centre (the load is assumed to be concentrated as this is the worst possible case); the formula for such a beam is $d = \frac{w\,l^3}{48\,\text{E}\,\text{I}}$.

Where d is the deflection at the centre,
w is the weight in lbs.,
l the length between supports in inches,
E is the modulus of elasticity,
I is the moment of inertia of the section; for a circular section $\text{I} = \frac{\pi}{64}\text{D}^4$,

where D is the diameter in inches. Substituting in the formula

$$w = 260 \text{ lbs.} = \cdot 116 \text{ ton},$$
$$l = 25''$$
$$\text{E} = 12{,}000 \text{ (the value usually taken for steel shafts)},$$
$$\text{I} = \frac{\pi}{64} \times 2^4 = \cdot 785,$$
$$\therefore d = \frac{\cdot 116 \times (25)^3}{48 \times 12{,}000 \times \cdot 785} = \cdot 004''.$$

And the radial depth of the air gap is $\frac{3}{16}'' = \cdot 1875''$.
The percentage deflection is therefore

$$\frac{\cdot 004}{\cdot 1875} = \cdot 022, \text{ or } 2{\cdot}2\%.$$

It is considered safe practice to make the shaft of such diameter that the deflection calculated by the above formula does not exceed 3% of the air gap.

The calculation of the magnetic pull due to a given displacement is not easy, the strength of field varies at all points round the armature, and the magnetic pull is therefore different at every point of the circumference. Only those components of the pull which are parallel to the line along which displacement has taken place are effective in bending the shaft, and it is therefore difficult to obtain an expression taking the whole of these considerations into account.

An approximation can be obtained by considering only those poles which are in a line with the displacement, and assuming that these act as flat faces with a uniform distribution of magnetic flux over them.

Thus, reverting again to the four-pole motor of Chapter IV., assume that the displacement is directly towards one pole and amounts to ·004″. The air gap at one pole will be decreased from $\frac{3}{16}'' = \cdot 1875$ to ·1835″; at the opposite pole it will be increased to ·1915″. The other two poles will give a side pull only, the forces will be equal and opposite, and need not, therefore, be considered.

Assume that the value of B varies inversely as the depth of air gap. The value of B, which is 7,000 lines per square centimeter for a uniform air gap, will become 7,100 at the pole face which the armature has approached, and 6,900 at the opposite pole. The pull is equal to $\frac{AB^2}{8\pi}$ dynes where A is the area of the opposing faces in square centimeters.

The pull will therefore be $\frac{260 \times (7{,}100)^2}{8\pi} = 502 \times 10^6$ dynes at one, and $\frac{260 \times (6{,}900)^2}{8\pi} = 489 \times 10^6$ at the other face, giving a total resultant pull of $502 \times 10^6 - 489 \times 10^6 = 113 \times 10^6$ dynes. But 981 dynes = one gramme and 1,000 grammes = 2·2 lb. ∴ 1 lb. $= \frac{981{,}000}{2 \cdot 2} = 445{,}000$

dynes. The pull due to a displacement of ·004″ will therefore amount to $\frac{113 \times 10^6}{445{,}000} = 254$ lb.

This should really be added to the weight of the armature in considering the shaft deflection, but the fact that the load has been considered as concentrated whilst it is really distributed along the shaft, and also the fact that the stiffening effect of the spider has been neglected, afford such a large factor of safety, that it is safe to set them off against the magnetic pull, and to leave this out of the formula.

§ 2. **The Bearings.**—The bearings of the smaller-size machines should have a length not less than three times the diameter of the shaft; in the larger sizes this may be reduced to 2½ diameters; in exceptionally large machines the bearing length is made even smaller than this proportion.

The bearings of almost all dynamo-electric machines are now made to be self-oiling, usually by means of ring lubrication. The pedestal is cast so as to form an oil reservoir of considerable capacity, and one or two brass rings are threaded on to the shaft and dip into this oil supply. The top half of the bush is slotted to receive these rings so that they hang freely on the shaft. When the shaft rotates, the rings rotate with it, and as they are dipping in the oil, they bring up a sufficient supply to keep the bearing well lubricated.

The oil after passing through the bearing is caught by a lip cast on the pedestal which leads it back into the reservoir, so that the bearings having once been filled need no attention for a considerable time. A cock is provided at the bottom of the pedestal to run out the oil, and at intervals when this has got dirty, the reservoir can be emptied, cleaned, and refilled with fresh oil. It is also convenient to have a gauge glass fixed to the side of the reservoir showing the level of the oil, since in case of any accidental leakage, the oil might fall so low that the rings

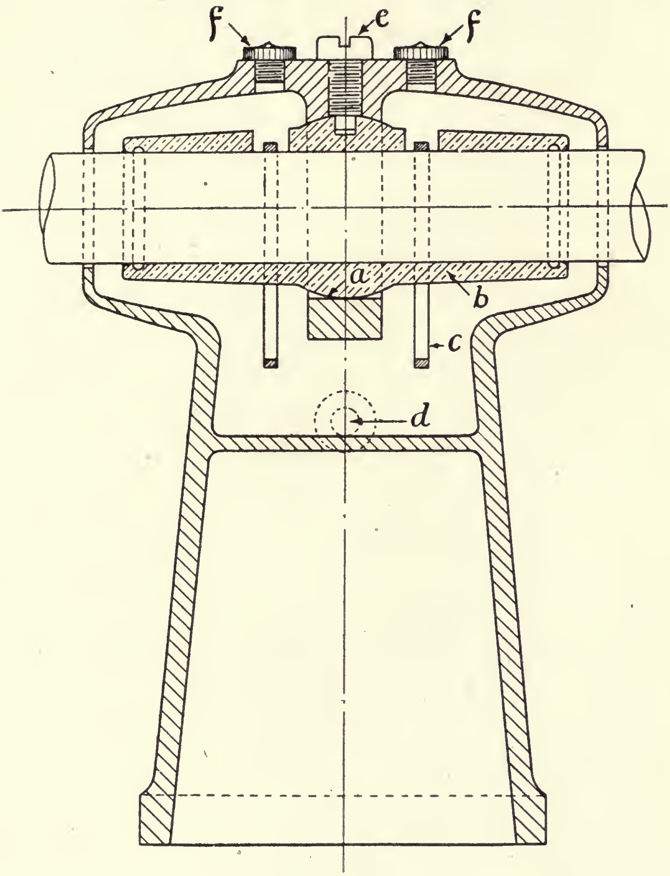


FIG. 53.

fail to reach its surface, in which case the bearing would of course run dry.

In the smaller sizes of machines the brasses are often cast with a spherical projection in their centre; they are supported on this spherical surface at the centre only, and are therefore free to adjust themselves through a small range to the position required to be in true alignment. Fig. 53 shows such a self-aligning, self-oiling bearing; *a* is the spherical projection on which the brass *b* rests, the oil rings are shown at *c*, the oil level at *d*. At *e* a pin is provided which enters a hole in the brass to prevent rotation and end movement; at *f* are sight holes for inspection of the rings to see that they are revolving freely. The use of self-aligning bushes is not adopted in machines of large sizes, where the bearing surface between the bush and its support becomes too small. It has been found in some cases where machines are subject to a considerable vibration, or to sudden stresses from large loads being suddenly thrown on, that self-aligning bushes are unsuitable, as the hammering spoils the outside surface of the bush and causes trouble; the brass gets too small to fit tightly in the pedestal, and the shake produced causes the armature to drop out of centre.

In alternating-current induction motors also, where the air gap is extremely small, it is usually found more satisfactory to put in solid bushes supported at both ends. In machines say over 24″ armature diameter it is good practice to use cast iron bushes filled with anti-friction white metal.

§ **3. Armature Spider.**—The spider consists of a cast iron hub which is keyed on to the shaft and on which the core discs and commutator are mounted. In small machines where space is limited this hub is solid, and turned to a cylindrical outer surface on which the discs are fitted (see Fig. 54). The common practice is to have one key between shaft and spider, and one between spider and discs; this latter key also registers the end plates, which consist of two castings between which the core discs are clamped.

In these small machines it is not unusual to secure the plates by means of a large nut screwing on to one end of the spider, and thus compressing the discs.

In large machines the spider hub is provided with

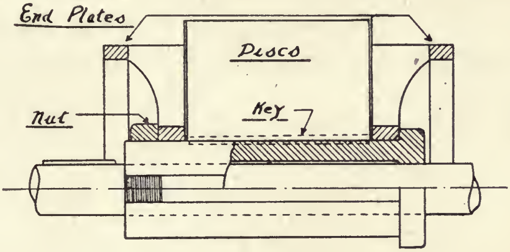


FIG. 54.

arms cast with it, the spaces between which are available for ventilation. In practice it is found that the minimum outside diameter of the boss is $1\frac{3}{4}$ times the diameter of the

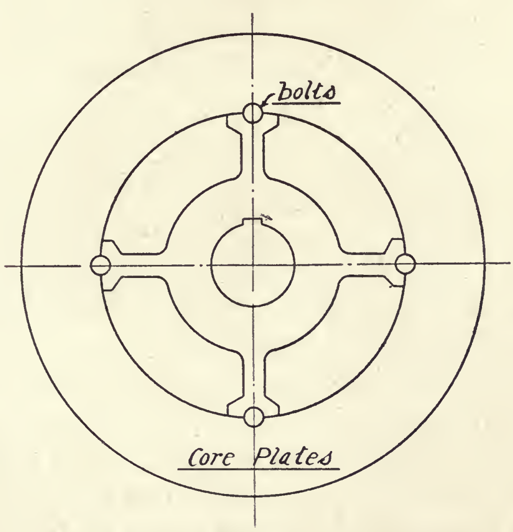


FIG. 55.

shaft; the difference between $1\frac{3}{4}$ times the diameter of the shaft and the bore of the core discs determines the length of the arms and the space available for ventilation.

In machines of such a size that an armed spider is available the discs are generally compressed between the end plates, not by means of a nut but by bolts, which may either pass half in the spider and half in the discs (Fig. 55), in which case they act as keys as well as clamping bolts, or which may pass clear of the discs between the spider arms (Fig. 56).

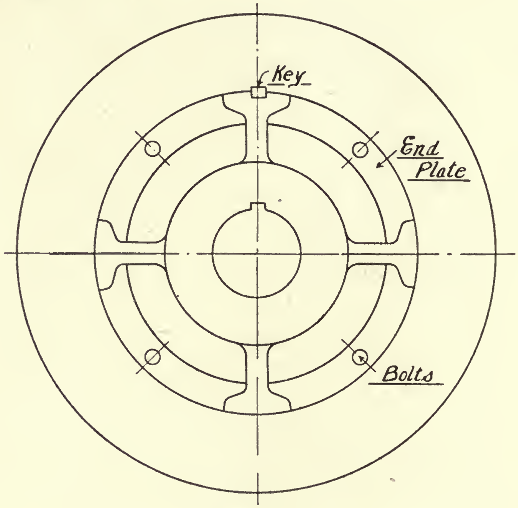


Fig. 56.

When the size of the armature becomes such that the discs cannot be obtained in one piece the core must be built up of segments. As it is necessary to secure these segments against centrifugal action, a clamping device has to be adopted to secure them to the spider, and this is accomplished either by dovetailed keys (see Fig. 57—the righthand half of which shows keys cast solid on the spider arms, the lefthand showing keys secured by means of bolts) —or by strengthening the end plates and passing bolts through the discs themselves (see Fig. 58).

In the latter case the depth of core for magnetic purposes should be reckoned not down to the core, but only to

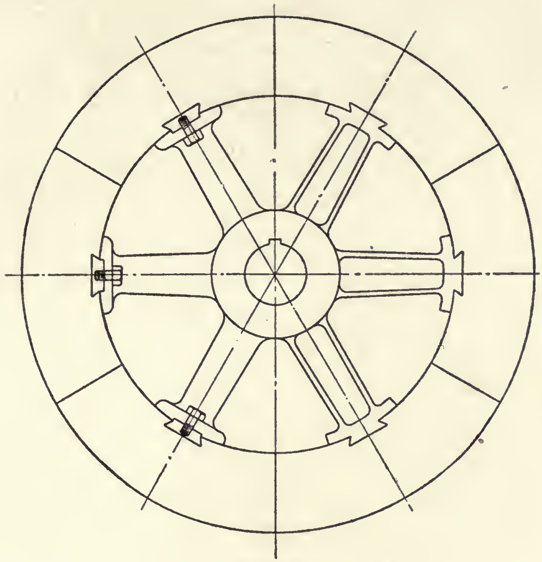


Fig. 57.

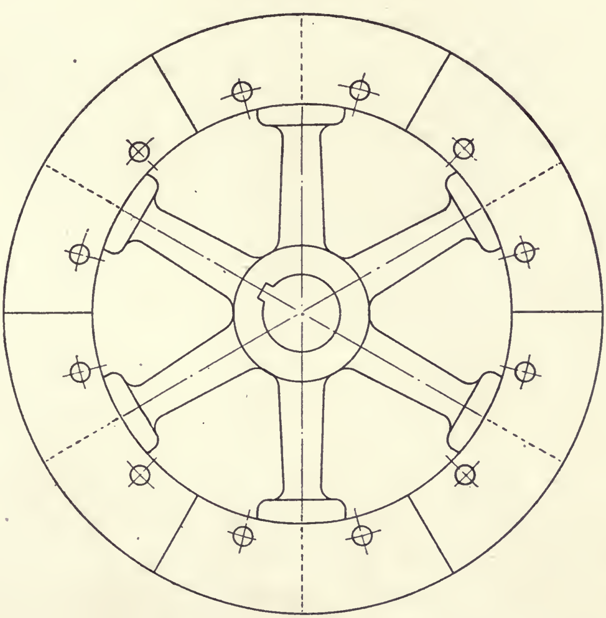


Fig. 58.

J

the top of the bolts. Of course in building up a core of this description, the segments must break joint; this requires careful watching of the number of bolts used, and of their position relatively to the slots.

§ 4. **Commutator Construction.**—One of the chief mechanical difficulties occurring in the construction of a dynamo is in the proper securing of the commutator parts. The commutator consists of a copper cylinder built up of comparatively narrow copper segments to which the armature windings are connected. Each of these segments must be insulated from its neighbours, and from all mechanical supports. Nevertheless, the construction must be absolutely rigid, as the slightest motion of any of the segments or of the insulating material between them spoils the cylindrical surface on which the brushes are running, and the slightest want of truth is apt to cause sparking at the brushes, which itself further spoils the surface, and, therefore, becomes worse and worse as time goes on.

Unfortunately no insulating substance has any great mechanical strength. In the best practice mica is the only substance used for the insulation of commutators. The copper segments are rolled to the proper taper, and on one side of each segment there is fixed with shellac varnish a sheet of mica of the same dimensions as the commutator segment and about $\frac{1}{32}''$ thick.

The section of the segment is easily calculated from the diameter of the commutator and the number of segments. If for instance a commutator is required having an outside diameter of 9″, and consisting of 165 parts, the circumference at the surface of the commutator will be $9\pi = 28{\cdot}3''$, and this divided by 165 gives for the width of each segment at the top ·171″; this includes the thickness of insulation between two segments. If this thickness be taken at ·03″, there is left for the copper a width of ·141″.

In order to allow room for the clamping arrangement, and also to allow a reasonable depth for wear before the commutator needs renewal, the depth of each segment should be not less than $1\frac{1}{2}''$. The diameter of the circle at the bottom of the segments will, therefore, be $6''$, the circumference will be $6\pi = 18{\cdot}9''$, and the width of each segment at the bottom with its insulation will be $\cdot 112''$. Again subtracting $\cdot 03''$ for insulation, there is left $\cdot 082''$ as the width of the copper segment at the bottom.

The segments having been rolled to this section and cut off to the proper length are then built up together with the sheets of mica between them, so as to form a cylinder of approximately $9''$ diameter. In this position they are clamped together, usually by means of a steel clamp, made in three or more pieces, which are put round the commutator and drawn tight, by means of bolts. The commutator in this condition is then put into the lathe and V-shaped grooves are turned at either end to receive the rings which, in its finished condition, are to hold the segments in place. When this operation is completed each segment is of the shape shown in Fig. 59.

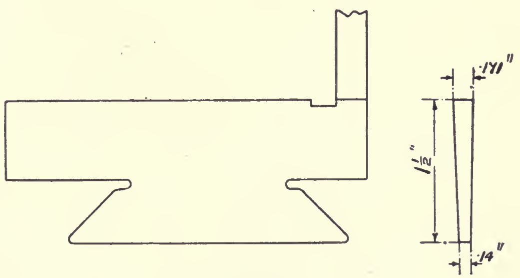


FIG. 59.

Usually the angle of the **V** groove is about 30°, the tendency having been to make this angle smaller and smaller. The usual practice some years ago was to make it 60°, but it was afterwards decreased to 45°, and now is very commonly, as we have said, about 30°.

The smaller the angle the greater the binding effect;

if, however, this is overdone the mechanical strength of the piece below the groove becomes too small; 30° has been found in practice to strike a mean between these requirements.

In the grooves so formed rings built up of mica or in

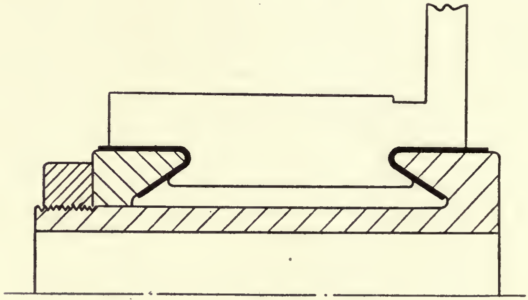


FIG. 60.

some cases of micanite are inserted. It is good practice to select the ring to be used for each end of the commutator, and to turn out the groove exactly to shape to receive

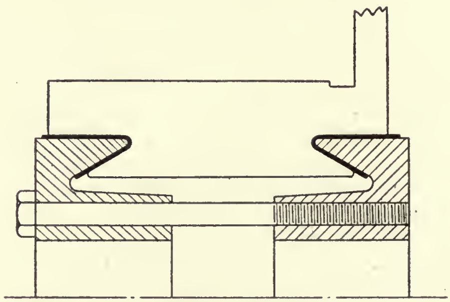


FIG. 61.

this ring, not working to a drawing, but to the actual dimensions of the particular ring to be fitted. Steel washers are then put on to each end of the commutator, fitting inside the mica ring, so as firmly to clamp the insulation into its place. The methods of fixing together and drawing in these two washers vary considerably in different practices.

Fig. 60 shows one of these washers as forming part of a sleeve which passes through the centre of the commutator. The other washer is threaded on to the sleeve, and is then tightened up by means of a large nut which is screwed on to the end of the sleeve. In Fig. 61 the two washers are not connected by any solid casting, but are simply drawn together by means of a number of bolts. In the case of this latter construction, the commutator when finished usually slips on to a cylindrical casting, which forms the commutator body and is part of the armature spider.

When made according to the former construction commutators were frequently put straight on to the shaft, which carried them quite independently of the armature core. It was found, however, that this method of carrying the commutator led to breakage of the connections between the armature and commutator. This has been a very frequent source of trouble, and was met by making the connections between commutator and armature winding more or less flexible so that they would adapt themselves to any slight relative motion between the commutator and armature core. Although this proved a remedy in many cases, the best practice and the safest method is certainly to mount the commutator on the same casting as the armature discs, so as to avoid any possibility of relative motion.

The actual shape given to the casting on which the commutator is carried varies largely in different makes of machines, and also with the size. Fig. 62 shows a commutator body made in the shape of an armed pulley, a construction frequently employed on large machines. When sufficient room can be obtained it is an advantage to leave the centre of the commutator quite open, so as to allow a free passage of air not only to cool the commutator itself, but also to obtain an easy access to the armature. Such a construction, however, allows dirt and foreign bodies to get into the commutator and reach the inner surface of the commutator segments, where they may cause a short-circuit between neighbouring parts with the result of burning out an armature coil.

Whatever method of construction is adopted the commutator should, when first put together, be heated to a considerable temperature, say something like 200° F.

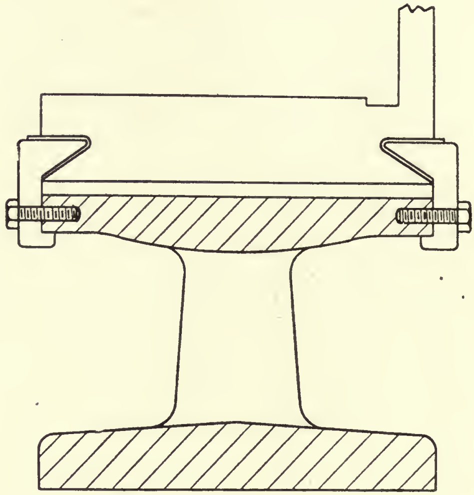


FIG. 62.

The bolts or nuts drawing the two washers together should be tightened whilst the commutator is hot, and again drawn up at intervals as it cools. The high temperature softens the insulation, and allows it, therefore, to be more readily compressed, but as the commutator cools, the copper contracts, and the commutator might, therefore, become slack if this contraction were not followed up by tightening again during the process of cooling.

§ 5. **Brush Holders.**—The brush holders used for holding carbon brushes on the surface of the commutator are made of many different patterns. They can, however, be divided into two classes—1. In which the carbon block is not rigidly held by any part of the holder, but is allowed radial motion and is pressed on to the surface of the commutator by a spring; 2. In which the carbon block is rigidly fixed in a part of the brush holder which is itself capable of motion, and is held down by a spring, so as to keep the carbon in contact with the commutator.

The requirements of a good brush holder are that it

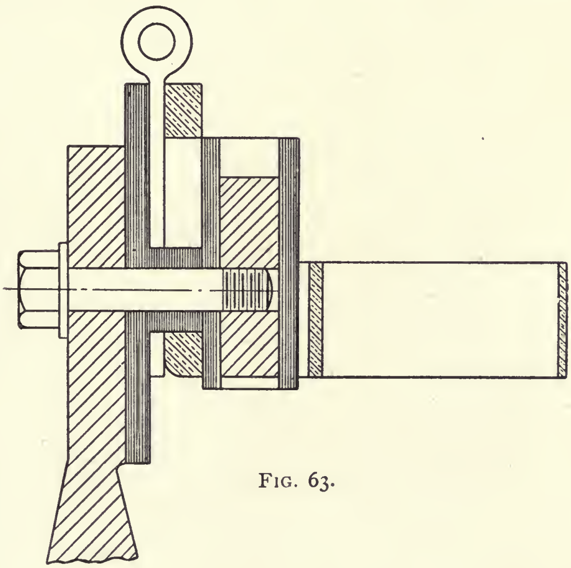


FIG. 63.

should keep the brush in contact with the commutator without necessitating an undue pressure which increases the friction losses. In order that small inequalities of the commutator face should not throw the brush off, the inertia of the moving parts should be as small as practicable, that is, the moving parts, whether the carbon block alone, as in Class 1, or the carbon block together with the part of the holder carrying it, as in Class 2, should be made as light as possible. In this respect Class 1 has the advantage over Class 2.

The brush should also be held in such a way that if it be thrown off the commutator for an instant, it shall fall back to exactly the same bearing. This necessitates that the carbon in brush holders of Class 1 should work between guides with a minimum of clearance, and in Class 2 that the pivots carrying the brush arm should have a long bearing surface so as to insure rigidity against side motion.

Again, the construction must be such as to insure that

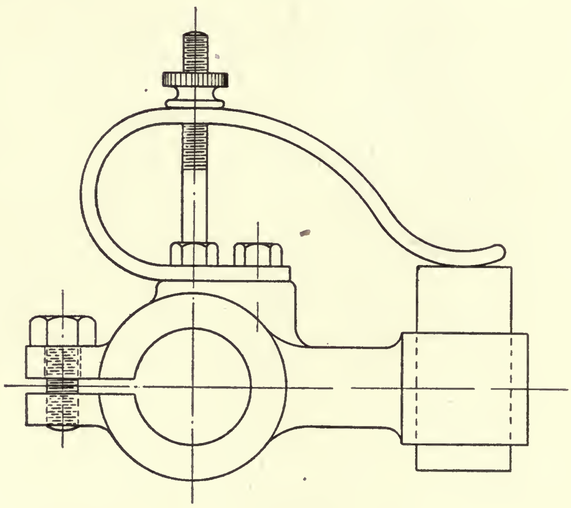


FIG. 64.

there is no sticking, and that the carbons are absolutely free radially to follow any irregularity of the commutator. In addition, provision must be made to carry the current through a positive connection to some fixed part of the holder, and no reliance should be placed on moving contacts such as the pivot in the case of a pivoted holder, or the contact between carbon and the box in which it moves in a holder of Class 1. This is usually provided for by fixing flexible copper wires on to the brush itself, and attaching these to some fixed portion of the holder. When this is done care must be taken that these connections are really flexible, and that they in no way impede the free motion of the brush. Figs. 63 and 64 show holders of Class 1, Fig. 65 a brush holder belonging to Class 2.

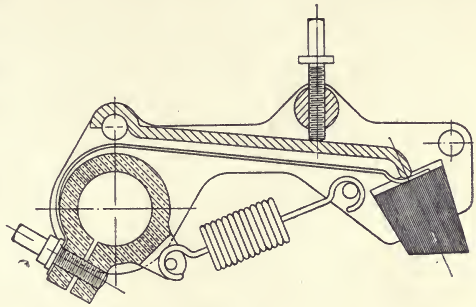


Fig. 65.

Brush holders for use with copper brushes are now seldom used except in special cases. Carbon brushes are not well adapted for use in machines which have to carry exceptionally large currents.

For instance, in machines made for plating and other electro-chemical work, very large currents are frequently generated at a small voltage. A machine may be required to give 2,000 amperes at an E.M.F. of 6 volts. The total output of this machine is only 12 kilowatts, and a machine of comparatively small size will easily do the work. But to collect 2,000 amperes, if carbon brushes were used, would require a commutator of very large size, in fact, amounting to several times the dimensions of the armature.

Again, in generators directly coupled to turbines, which work at a very high speed, the current is also large compared with the size of the machine, because a high speed allows the general dimensions of the machine to be kept down. The commutator, however, is not affected in this way, and would again have to be of very abnormal dimensions if used with carbon brushes. The difficulty in this case is aggravated by the fact that the high peripheral speed at which the commutator runs makes the collection of the currents in any case difficult.

There are shown in Figs. 66 and 67 some types of brush holders adapted for use with copper brushes. In another type specially arranged for use on high-speed turbo-generators springs are entirely dispensed with, and the brush is

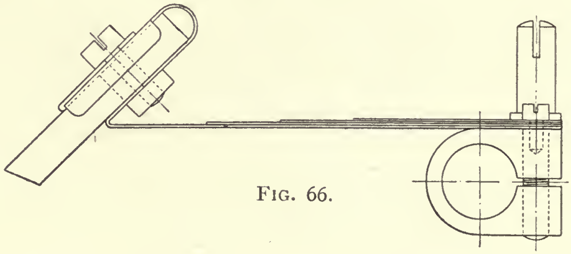


FIG. 66.

kept down to its work by means of a weight. Other holders of a very elaborate nature have been devised to allow of carbon brushes being used on such high-speed commutators. The pneumatic holder is an example of these, where the

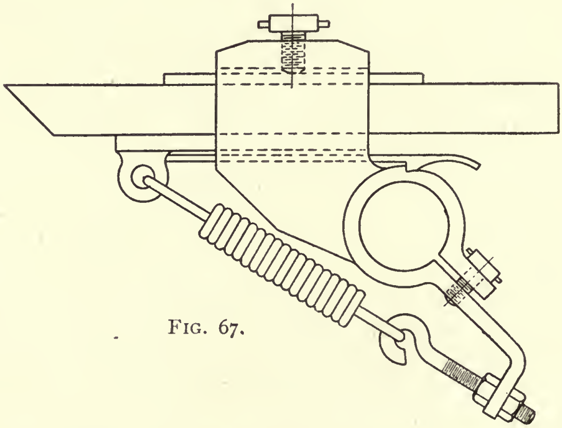


FIG. 67.

pressure is kept on the brush by means of compressed air.

Whatever the class of brush holder, it must be mounted so that the brushes are capable of circumferential adjustment round the commutator.

Even if the machine is specified to run with a fixed brush position, it is necessary that the best brush position should be found on test, and in order to enable this to be done

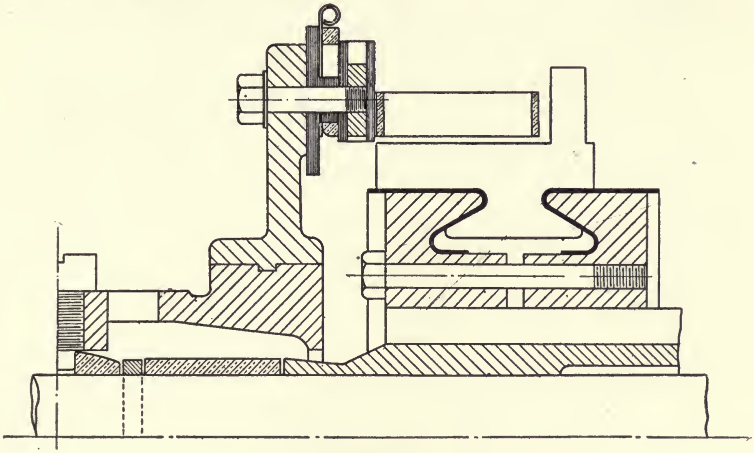


FIG. 68.

conveniently the brush holders should be mounted on a rocker. The rocker is a casting mounted on the bearing pedestal and carrying all the brushes; it is usually made in two halves bolted together; by tightening the bolts after the correct brush position has been found further motion of the brushes is prevented (Fig. 68).

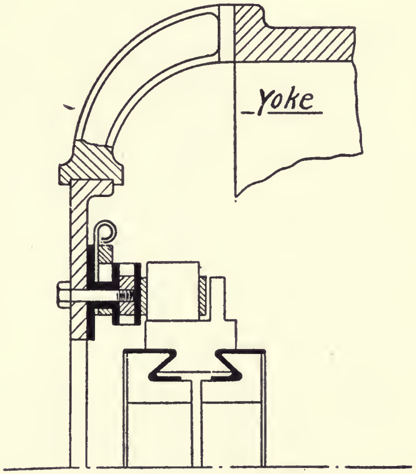


FIG. 69.

In large-size machines the brush holders are fixed to a large ring carried by brackets from the magnet yoke instead of being mounted on the pedestal (Fig. 69).

In the case of all large machines a hand wheel is provided which by means of a worm motion gives a slow movement of rotation to the rocker. In smaller machines no such adjustment is provided, and the rocker is moved by hand.

On small enclosed motors the brush holders are sometimes carried not by a special casting, but by the end cover of the motor which also carries the bearing. If this is done it is advisable that the bolts holding the end cover on the shell should pass through slots in the cover so as to allow it to be rotated through a small angle, thus allowing the best brush position to be found; when this is accomplished the bolts are screwed up and the cover firmly fixed in place.

The brush holder must of course be insulated from the body of the machine. On continuous current dynamos this is done by insulating the spindle or box carrying a line of brushes from the rocker. In some polyphase machines provided with slip rings, brushes of different polarity and running on different rings are sometimes mounted on the same spindle; it is then necessary to insulate each brush holder from the spindle which carries it.

CHAPTER VI

CONTINUOUS-CURRENT DYNAMOS AND MOTORS FOR SPECIAL PURPOSES

DYNAMOS have at times to be designed to meet special requirements which necessitate departure from the usual lines of design. A few such cases will now be noticed.

§ 1. **High-Speed Generators.**—Since the steam turbine has become of practical importance, special attention has been directed to generators designed to run at a very high speed. High speeds are necessary to give good efficiency in the steam turbine, and it is not uncommon to run as high as 3,000 revolutions per minute for outputs up to, say, 3,000 H.P. Generators to be driven by such turbines have to be coupled directly to the turbine and must, therefore, run at the same speed.

The difficulties arising in the design of continuous-current dynamos to run at such high velocities are chiefly difficulties of commutation and mechanical difficulties due to the centrifugal force at a high peripheral speed.

The reactance voltage of such machines is necessarily high, because it is obvious that the armature should be made of comparatively small diameter, and therefore of comparatively great length in order to avoid dangerously high peripheral speeds. In the case of alternators, the magnets can be made the rotating part of the machine, and their construction may be such as safely to allow of comparatively high peripheral speeds; but a continuous-current armature, carrying conductors on its periphery which must be secured against centrifugal forces, does not lend itself to peripheral speeds greater than 10,000 to 15,000 ft. per minute as an outside limit. It is thus seen that an armature running at, say, 3,000 revolutions per minute, cannot have a diameter greater than about 18″, and if a large output is required, the armature must be made of comparatively great length.

Thus if a generator is required to run at 3,000 revolutions, and give an output of 2,000 kilowatts, the value of D, the watts per revolution, will be $\frac{2{,}000{,}000}{3{,}000} = 670$, the corresponding $d^2 l$ is about 15,000, and the diameter must not exceed 18″, therefore l will be, say, 46″.

Compare this with a machine to give 333 kilowatts at 500 revolutions, the value of D and therefore of $d^2 l$ will be the same, but the diameter can now be increased, and the values of d and l may be taken at, say, 30″ and 13″. The difference in the reactance voltages will be apparent on substituting in the formula on page 81. In the first case the pole area taking 4 poles will be $\frac{18\pi}{4} \times \cdot 7 \times 46 = 455$ square inches, and the value of N working at a density of 7,500 lines per square centimeter in the air gap will be $455 \times 6{\cdot}45 \times 7{,}500 = 22 \times 10^6$; in the second case, taking 6 poles, the pole area will be $\frac{30\pi}{6} \times \cdot 7 \times 13 = 286$ square inches, and the corresponding value of N is $286 \times 6{\cdot}45 \times 7{,}500 = 13{\cdot}8 \times 10^6$.

In each case there will be one turn only per commutator part, and the reactance voltage will be

$$\rho = \frac{2{,}000{,}000 \times 20 \times 1 \times \pi \times 46}{4 \times 22 \times 10^6} = 66$$

$$\text{and } \rho = \frac{333{,}000 \times 20 \times 1 \times \pi \times 13}{6 \times 13{\cdot}8 \times 10^6} = 3{\cdot}25$$

respectively.

This value is very much too high in the case of the high-speed machine to give satisfactory results, and it is therefore usually found necessary in such machines to employ some of the special commutating devices described at the end of this chapter.

§ 2. Variable-Speed Motors — Speed Variations by Series Resistance.—Difficulties of the same nature arise in the case of continuous-current motors which are required to run at a variable speed. Variation in speed can be readily obtained in two different ways. A resistance may be introduced in the armature circuit: this reduces the voltage at the brushes, and thus decreases the speed. This method is frequently used in conjunction with series-wound motors on such apparatus as cranes and for driving vehicles, as, for instance, tramcars, and it is well adapted to this purpose. It suffers, however, from two serious disadvantages.

Firstly, it is extremely inefficient, as a large amount of the power drawn from the mains is wasted in the resistance. In fact, if it can be assumed that the torque required by the driven apparatus is constant (this is approximately true in many instances), the current drawn from the mains is the same at whatever speed the motor is run. If, for example, the apparatus is to be made to run at half speed, sufficient resistance must be introduced to absorb half the voltage. The current flowing through the motor and resistance is the same as it was at full speed, but of the watts drawn from the mains half only are used in the motor, the other half are wasted in the resistance, and the efficiency of the combination can therefore never rise to as much as 50%.

Secondly, in addition to this very serious drawback, a further one is found in the fact that the speed at which the motor runs is dependent not only on the actual amount of resistance in the circuit, but also on the current flowing in that resistance, so that a change of load causes a corresponding change of speed unless suitable change is made in the amount of resistance. For many purposes this is a serious disadvantage, and as a matter of fact this method of speed regulation is never used except in cases such as those mentioned above, where an attendant is constantly at hand to regulate the speed at every instant as may be required.

§ 3. **Speed Variation by Shunt Resistance.**—A second method of speed control for a continuous-current motor is applicable to shunt-wound or compound-wound motors, and consists of introducing a resistance in series with the shunt winding. Since the shunt current is only a small percentage of the current flowing in the motor, the energy lost in a shunt resistance is inconsiderable, and therefore this method is greatly superior to the former one on the score of efficiency.

In a shunt-wound motor for any given position of the resistance switch, the speed will be approximately independent of the load. The effect of introducing resistance into the shunt circuit of a motor is of course to decrease the current in this winding, therefore to decrease the number of ampere-turns on the machine and consequently the total magnetic flux. The back E.M.F. generated by the motor is proportional to N and to the speed. On introducing resistance, this back E.M.F. is diminished, since the value of N is decreased. The E.M.F. applied to the terminals of the motor is, however, constant, and if the back E.M.F. falls, a greater current will flow through the armature. This will cause it to accelerate, and the speed will increase until the back E.M.F. is again brought up to a value corresponding to the E.M.F. applied at the terminals. A numerical example will make this clear.

The motor designed in Chapter IV. normally runs at 600 revolutions per minute, and has a total magnetic flux per pole (N) of $1 \cdot 8 \times 10^6$ lines. If resistance be introduced in the shunt circuit the number of ampere-turns per pole will be decreased. Assume that resistance is introduced to such an amount as will reduce N to $1 \cdot 2 \times 10^6$ lines. The back E.M.F. generated will now be

$$\frac{660 \times 4 \times 1 \cdot 2 \times 10^6 \times 600}{60 \times 10^8} = 318,$$

instead of

$$\frac{660 \times 4 \times 1 \cdot 8 \times 10^6 \times 600}{60 \times 10^8} = 471$$

(see formula on page 53).

The resistance of the armature circuit is 1·6 ohm, and since the E.M.F. of the mains is 500 volts, there will be available $500 - 318 = 182$ volts, and the current will be $\frac{182}{1 \cdot 6} = 115$ amperes. The full-load current of the machine is only 18 amperes, and the much greater torque due to the increased current will cause the armature to accelerate rapidly.

It should be noted that even if the change in the shunt resistance is made suddenly the field strength will not adapt itself instantly to the new conditions. The change will be gradual and acceleration will begin to take place as soon as the current exceeds 18 amperes. By the time the field has settled down to its new value the speed and therefore the back E.M.F. will have increased, and a current of 115 amperes will not actually flow in the armature. This figure merely represents the highest limit which the current would attain if the field strength could be varied instantaneously, and the actual current will approach more or less nearly to this upper limit according as the change of field strength is less or more rapid.

If it may be assumed that the torque required by the load is independent of the speed, 18 amperes will still be required in the armature conductors to drive the load, and acceleration will go on until the back E.M.F. again reaches 471 volts (since $18 \times 1{\cdot}6 = 29$ volts are dropped in armature resistance and $500 - 29 = 471$). With the weakened field the speed required for this is given by the equation

$$\frac{R}{60} = \frac{471 \times 10^8}{4 \times 1{\cdot}2 \times 10^6 \times 660} = 14{\cdot}5$$

(a simple modification of the equation on page 53).

$$\therefore R = 14{\cdot}5 \times 60 = 870 \text{ revolutions per minute.}$$

It is evident from the above that the motor will run at its lowest speed when the full shunt current is allowed to pass, and that the speed will increase as the resistance is introduced into the shunt circuit. That is to say—the motor must be designed for the lowest speed at which it is intended to run it, and must, on that account, be considerably larger than would be the case if it had been got out for a high speed only. This fact is specially noticeable in cases where the higher power is required at the higher speed only and the power at the lower speed is considerably reduced.

Take, for instance, a shunt-wound motor intended to drive a fan at 1,200 revolutions per minute, the H.P. required being 10, and intended also to run at 600 revolutions per minute when the necessary H.P. is reduced to 2. The size needed for such a machine will be very nearly that of a motor capable of giving not 2 H.P., but 10 H.P. at 600 revolutions per minute. The number of bars on the armature, and the number of magnetic lines per pole, must be such as to give a speed of 600 revolutions. Nevertheless, the bars must be large enough to carry safely the current required for 10 H.P., since this current will be put through when the fan is running at 1,200 revolutions, and this method, therefore, whilst much superior to regulation by series resistance, has the disadvantage of requiring a larger motor, and therefore entails an increased first cost.

Having settled the size of the motor, the difficulties met with, in the design, are due to the fact that at the high speed, when a large current is taken by the motor, the value of N is at its lowest. Inspection of the formula for reactance voltage will show that the reactance voltage varies inversely as the value of N, and if, for instance, this value be made half of the maximum in order to obtain double speed the reactance voltage will thereby also be doubled. To keep this value within reasonable bounds, the size of the motor must be still further increased, and although, in a few instances, motors with even a larger speed variation than 2 to 1 were put on the market, it was seldom considered advisable to attempt even a 2 to 1 variation, because of the largely increased size of motor required. Of recent years, however, the introduction of commutating poles, one of the special devices to be described later, has made it possible to work with very high reactance voltages, and it is now practicable to get speed variations of even 3 or 4 to 1, without prohibitive increase of size.

§ 4. **Series-Parallel Control.**—When two motors are used on the same load, as, for instance, the two motors driving a tramcar, the method of varying the speed known as the series-parallel control is often used. A controller or switch is used which alters the connection of the motors to the mains so as to put them either in series or in parallel with one another.

The motors are each designed to give normal speed at, say, 500 volts. When they are coupled in parallel across the mains they will run at this speed, but when the connections are altered so as to put the motors in series, each will have on its terminal only 250 volts, instead of 500, and will run at approximately half speed. Fig. 70 shows the connections of the motors to the mains when in series and when in parallel.

This method gives only two speeds, full and half, and

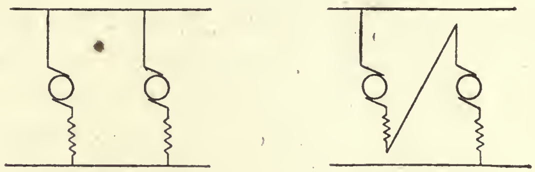


FIG. 70.

if intermediate speeds are required, either a series or shunt resistance must be used in combination with the series-parallel controller.

In cases where only one motor is used the series-parallel method may be employed by putting on the armature two independent windings, each connected to a separate commutator; the controller then connects the two commutators either in parallel or in series across the mains, and speed regulation is obtained in exactly the same way as with two motors.

§ 5. **Motors for Intermittent Work. Rating of Crane and Traction Motors.**—So far difficulties have been found in the special cases considered on account of commutation troubles. In the cases now to be considered special designs are required from the point of view of heating.

It has been pointed out in Chapter IV. that the rated H.P. of a motor is settled not by the fact that no greater horse-power can be got from it, but by the largest current which it will satisfactorily carry continuously.

But if the motor is only required to work intermittently, it can in many cases be rated at a much higher power than would be permissible for continuous work. Crane motors and traction motors are typical cases of machines designed for intermittent work. The hoisting motor fitted on, say, a 10-ton travelling crane, will not be at work for more than a small portion of the time. If the crane is fitted in a general engineering shop, it will probably have to hoist 10 tons only at very rare intervals, and will be engaged, generally, handling very much smaller weights. Although it must therefore be capable of sufficient H.P. to deal with the heaviest weight, it need only be capable of doing this maximum H.P. without sparking; the question of the temperature-rise is comparatively unimportant.

The same argument is true of a tramcar motor which, whilst it must be capable of a considerable effort for a short period in order to accelerate the car on the stiffest gradient, is usually running at a very much smaller output. Such motors are therefore made very much smaller than motors for general work required to give the same H.P., that is to say, a motor which would ordinarily be called a 10 H.P. motor may, for crane or traction work, be rated at 15 or even 20 H.P., and provided care is taken that its constants are such as to allow it to give this higher power without sparking, the results are quite satisfactory.

The question of the temperature-rise allowable in such cases has not yet been altogether standardised in Great Britain. The rating frequently used for tramcar motors was imported from the United States, and is to the effect that the motor on test shall not rise more than 100 degrees Centigrade when run for one hour at its rated full load. This, of course, does not mean that the temperature in actual practice would ever rise to this amount, which would cause deterioration of the insulating material, but that it has been found as a matter of usage that motors which do not exceed this rise on a test of one hour's duration do not get dangerously hot under the actual working conditions on a day's run.

A similar standard has been discussed for use with crane motors, but the conditions under which different cranes are employed are more dissimilar than is the case with different traction systems. It has, therefore, been proposed by some manufacturers that the rating of the motor should be based on lines as nearly as possible similar to the working conditions. They have thus proposed that the motor should be rated at such a power that, when run on test for a specified number of minutes, say two, four, or six minutes out of every twelve, it shall not rise more than 70° F., however long the process be continued. This method, whilst theoretically good, means a tedious and troublesome test, and, probably for this reason, has not been very generally adopted. Crane-makers more usually specify a test of either half or one hour's duration at full rated load, the temperature-rise at the end of this run not to exceed either 70° or 100° F.

If these are taken as the standard crane ratings at present generally used in Great Britain, it will be seen that for any given H.P. required, any one of four different sizes of motors may be used according to the specification. The smallest will be the half-hour rating with 100° F. rise, whilst the largest will be the one rated for one hour at only 70° F. rise. Which of these four ratings should be used in any special case depends upon the conditions under which the crane will be generally worked.

§ 6. Commutating Poles.—Amongst other devices which have been brought out for the purpose of improving the commutation, both in motors and generators, that known as commutating poles or interpoles is now coming largely into use. In order to effect the reversal in the armature coil which is short-circuited by the brush, an external

E.M.F. is required which, if it is to effect a perfect reversal at all loads without change of brush position, should be proportional to the load the machine is carrying. In an ordinary machine this E.M.F. may be supplied by the magnetic field in the interpolar space.

Unfortunately, as has already been pointed out, it is necessary, in order to get into a field having the right polarity to effect reversal, to move the brushes forward in the case of a generator, backward in the case of a motor. In both instances, the motion is towards that pole tip which is weakened by the reaction of armature, and since the weakening effect of the cross turns depends on the current the armature is carrying, this field will at any one given point get weaker as the load increases, that is, the field will be weakest just at the time when the strongest reversing force is required. It will therefore be necessary to move the brushes nearer the pole so as to get into a region of stronger field as the load increases. As it is quite usual to specify that a machine shall work from no-load to full load without change of brush position, this method of compensating for the weakened field at full load is not available, and in a machine which is not supplied with any special commutating device, it is necessary that the reactance voltage be kept so low that good commutation is obtained without the assistance of an external reversing field. The brushes can then be placed on or about the neutral axis, and remain there for any load.

A very low reactance voltage is only obtained at considerable cost in the construction of the machine. The number of commutator parts must be abnormally large, and the machine has not infrequently to be of larger diameter and shorter length than would be dictated by considerations of cost of manufacture. In the case of motors where a large variation in speed is required, by means of shunt regulation, the problem becomes still more difficult, and an increase of speed of more than 100% is practically impossible on an ordinary machine.

It has already been pointed out that a series-wound motor is superior to a shunt-wound in respect of commutation, from the fact that the strength of the whole magnetic field, and therefore also of the fringe in which commutation is taking place increases with increase of load.

A commutating pole consists of a magnet core and pole shoe fixed in the gap between two main poles (see Fig. 71), the winding of which is connected in series with the armature, and the object of this pole is to provide a magnetic field, such that the motion of the conductors under the brush will generate an E.M.F. of a direction and strength suitable to effect sparkless reversal even with a high value of the reactance voltage. The winding of the reversing pole being in series with the armature, the strength of field produced will be proportional to the load, and thus, if the right proportion of turns is found to give a good reversal at any one value of the current, the same relative strength of field will be present at all other values of the current, and the commutation will be sparkless at all loads without any change of brush position. The only limit to this is the saturation of the steel in the auxiliary pole. When this becomes saturated, increased current does not produce a correspondingly increased field, and the armature reaction

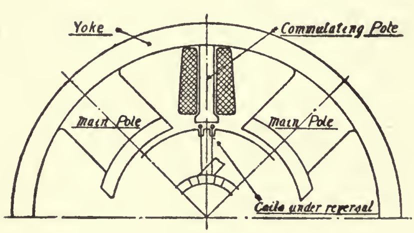


FIG. 71.

being increased, the proper equilibrium between the reversing field and the self-induction of the short-circuited coil no longer exists. Fig. 72 shows diagrammatically the connections of a four-pole shunt-wound machine having

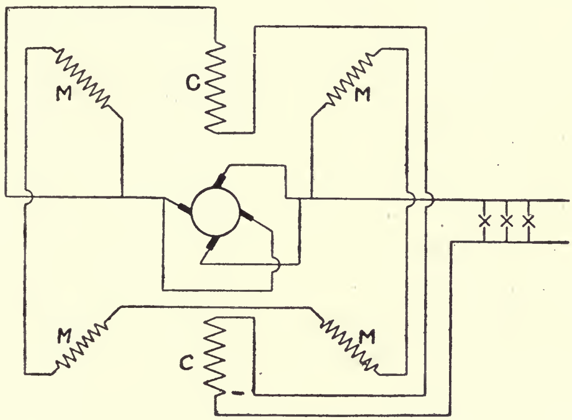


FIG. 72.

two commutating poles. M M represent main, and C C commutating poles.

In calculating the number of ampere-turns required on a reversing pole, it must be noticed that the whole of the armature ampere-turns are back ampere-turns, as far as this point of the armature is concerned, and a number of ampere-turns equal to the armature ampere-turns per pole must, therefore, be put on in addition to the turns required to put the reversing flux through the air gap and through the metal of the pole. The number of ampere-turns thus found appears at first sight to be considerable, but the fact that only a comparatively small flux is required enables the pole to be made of small dimensions. The mean turn is therefore small, and the amount of copper required for the winding is, in most cases, not very large.

The calculations of the amount of reversing flux required are based on the assumption that the reactance voltage of the machine is known and that an E.M.F. sufficient exactly to counterbalance the mean value of the reactance voltage should be generated in the short-circuited coil by the reversing flux. This flux may be obtained in several ways. The area of the auxiliary pole face may be varied, and the magnetic induction in the air-gap may also be varied.

It is well to work at a fairly high density; the number of ampere-turns required to overcome the gap reluctance is in any case not a very large proportion compared with the number of ampere-turns required to overcome the armature back turns. The area of the pole face can be suitably modified; the width of the auxiliary pole should be not less than one slot and one tooth, the length measured parallel to the shaft may be cut down to any desired extent provided the required area is obtained. Whilst in an ordinary machine the reactance voltage cannot be allowed to exceed two volts if good commutation is to be obtained with a fixed brush position, with suitable interpoles the reactance voltage may be allowed to increase up to 10 or 12 volts without inconvenience.

The adoption of reversing poles is very effective for working satisfactorily with such high reactance voltages, and is therefore extremely useful for special cases such as were noticed at the beginning of this chapter, where it is impossible to get out a design which will keep the reactance voltage down to the usual limits. Also, in some cases where a design on the usual lines could be quite readily got out, it is possible by the use of reversing poles considerably to lessen the depth of the air gap, and therefore the weight of copper required, and thus in various ways to cheapen the machine. But, however well proved the advantage of commutating poles may be in these cases where special commutating devices are necessary, it is doubtful whether it is sound policy to use them, whether absolutely necessary or not, throughout a line of machines many of which could quite well be designed without their use.

A numerical example will now be given, showing the calculations required for finding the necessary flux and the necessary number of ampere-turns on the commutating pole. The reactance voltage is first calculated by means of the formula on page 81.

If it be required, for instance, to increase the speed of the motor designed in Chapter IV. from 600 to 1,800 by means of shunt resistance, this will require that the field strength should be reduced to one-third; namely, to N = 600,000, and the reactance voltage instead of being

$$\rho = \frac{500 \times 18 \times 4 \times 20 \times \pi \times 6}{4 \times 1{\cdot}8 \times 10^6} = 1{\cdot}83,$$

as given on page 96, will become:—

$$\rho = \frac{500 \times 18 \times 4 \times 20 \times \pi \times 6}{4 \times {\cdot}6 \times 10^6} = 5{\cdot}5.$$

This is the maximum value of the reactance voltage on the assumption that the variation of strength in the coil under the brush is represented by a sine curve; on this assumption the mean value of the reactance voltage will be $\cdot 636 \times 5{\cdot}5 = 3{\cdot}5$ (page 164), and it will be necessary that the commutating pole should give such a flux as will generate in the short-circuited coil an E.M.F. of 3·5 volts.

Either two or four commutating poles may be used; a little consideration of the winding diagrams given in Chapter II. will show that even if only two commutating poles are used, all the coils which are undergoing commutation will have some of their conductors under one or other of these poles. If two poles only are used, the number of conductors belonging to one coil which will be in a commutating field at one instant will be four, since there are four turns per coil. It is required to generate in the coil an E.M.F. of 3·5 volts, each conductor must therefore have induced in it an E.M.F. of about ·9 volts, that is, $\cdot 9 \times 10^8$ C.G.S. units of E.M.F., that is, it must cut lines of magnetic force at the rate of $\cdot 9 \times 10^8$ lines per second. The peripheral speed of the armature is $12\,\pi \times 600 = 22{,}700$ inches per minute, which is equal to $\frac{22{,}700 \times 2\cdot54}{60} = 960$ centimeters per second. If, then, the magnetic field be such as to give $\frac{\cdot 9 \times 10^8}{960} = 94{,}000$ lines in each centimeter, measured circumferentially, the rate of cutting lines will be that required to give 3·5 volts in the short-circuited coil.

If the pole shoe of the commutating coil be made 12 centimeters long, measured parallel to the shaft, the magnetic density in the air gap will be $\frac{94{,}000}{12} = 7{,}800$. This is a suitable density to work at. The width of the pole should be such that the coil remains in the field during the whole time of short circuit, say $1\frac{1}{4}''$ in this case. From which it follows that the pole shoe should be $1\frac{1}{4}''$ wide, measured circumferentially, and 12 cms. $= 4\frac{1}{2}''$ long, measured parallel to the shaft. If four commutating poles were used instead of two only, each could be made half the length, since each pole would be required to generate only half the E.M.F.

The total flux from one pole will be found by multiplying the area of the shoe in square centimeters by 7,800, the value of B. The area $= 12 \times 3\cdot2 = 38\cdot4$ square centimeters, and the total flux $N = 38\cdot4 \times 7{,}800 = 298{,}000$ lines. The magnet core must be made of sufficient section to carry these lines, and a large coefficient of leakage must be allowed for, because whilst the leakage paths are not materially different from the leakage paths of the main poles in their sectional area, the total flux from the commutating pole is very much smaller, and therefore a much larger proportion of the total magnetic lines will pass along the leakage paths. A coefficient of 2 may be used; the number of lines in the magnet coil will then be 298,000 $\times 2 = 596{,}000$, and working at about 13,000 lines per square centimeter, the area of the steel must be $\frac{596{,}000}{13{,}000}$ $= 46$ square centimeters, or 7 square inches. A round magnet core 3″ diameter will give very nearly this area.

The number of ampere-turns to be provided on each pole are equal to the armature ampere-turns per pole added to the ampere-turns required to put the magnetic flux through the air gap. The number of wires on the armature is 1,320, the number of turns $\frac{1{,}320}{2} = 660$, and the number of turns per pole $\frac{660}{4} = 165$, and each turn carries at full load 9 amperes; the armature ampere-turns per pole are therefore 1,485, say 1,500, and if the depth of the air gap under the commutating pole is made $\frac{3}{16}$″, the same as under the main poles, the ampere-turns required will be $\cdot 475 \times 7{,}800 \times \cdot 8 = 2{,}900$, say altogether 4,500 ampere-turns, and at full load 18 amperes is the current carried by the motor. The number of turns on each commutating pole must therefore be $\frac{4{,}500}{18} = 250$. The two commutating poles will be connected in series with one another and with the armature.

That the commutating pole winding should be connected in series with the armature is, as already pointed out, the essential feature of this device, as it insures the value of the commutating field being proportional to the current it has to reverse. Commutating poles are also referred to as interpoles, auxiliary poles, and reversing poles.

§ 7. **Compensating Coils.**—Another device with a similar object consists of compensating coils connected in series with the armature, and let into slots in the main pole faces. They are so connected as to oppose the armature reaction. These coils cause the reversing flux at the tip of the main pole face to remain comparatively unchanged in value under considerable fluctuations of armature current, and in this way assist in obtaining good commutation without change of brush position. Whilst the commutating poles produce an entirely new field in which the reversal is carried out, the object of the compensating coils is to maintain the fringe of the main field at such a strength as is suitable for sparkless reversal.

In order to obtain perfect compensation, the number of conductors carried across each pole face must be such as to give a number of ampere-turns equal to the armature ampere-turns per pole; if this number be provided, the armature reaction will be entirely compensated for, the current in each armature conductor will be neutralised by an equal current flowing in the opposite direction in the compensating coils, and the field strength will therefore not vary, whatever the load on the machine. It is evident, however, that a considerable number of watts will be lost in such a winding, and not only will the efficiency suffer, but the heating will be increased, unless the dimensions of the machine are altered so as to provide increased cooling surface.

§ 8. Sayer's Commutator Coils.—In both the above instances it will be seen that the object sought is attained by introducing an E.M.F. in the short-circuited coil by means of a magnetic field. In the commutating device known as Sayer's coils, the same object is attained by providing on the armature special conductors, which are brought into play only during reversal.

The main winding of Sayer's armature is exactly similar to that of any other machine, but it is not connected directly to the commutator; instead it is connected to a special conductor which is brought back along the armature core so as to cut the magnetic field and is then connected to a commutator part. This conductor is therefore idle except when the brush is in contact with the commutator part to which it is connected. At other times it may be cutting the magnetic field and having an E.M.F. induced in it, but since it has one end free, and disconnected from all other conductors, no current can pass through it. When the brush causes current to flow through the commutator coil, the E.M.F. then generated in it will be in such a direction as to help to reverse the current in the short-circuited coil. Fig. 73 shows diagrammatically the connections from part of the armature winding to the commutator in a Sayer's wound armature.

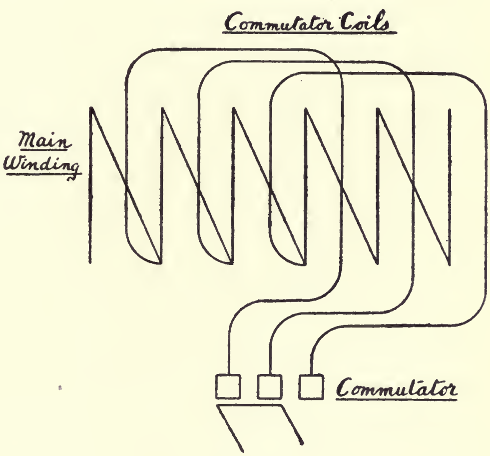


FIG. 73.

The chief disadvantages of this method are that it takes up a large amount of space on the armature just where space is most valuable. Secondly, that it increases the self-induction of the short-circuited coil of the armature by adding to its length two new conductors, themselves generally embedded in slots, and the self-induction of which is therefore considerable.

The one great advantage it possesses is that the direction of the commutator coils can be made such as to cause the reversal to take place under the leading pole tip in the case of a generator, and the trailing tip in the case of a motor. That is to say, Sayer's winding can be arranged so that the brushes are brought backwards in the case of a generator, and forward in the case of a motor, instead of the opposite arrangement as in an ordinary winding. The result of this is not only that the reversal takes place in both cases under the strengthened instead of under the weakened tip, but it also has the effect of changing the armature ampere-turns from back ampere-turns to forward ampere-turns. Thus in both cases the armature ampere-turns will strengthen the field as the load increases, instead of weakening it, and a generator wound on this principle tends to compound and maintain a fixed E.M.F. under all fluctuations of load.

In practice it was generally found that this effect was not sufficient to keep the voltage constant with increasing load, and that a few series turns had to be added.

CHAPTER VII

ALTERNATING CURRENTS

§ 1. Representation of Alternating Currents by Sine Curves.—An alternating current is one which, instead of flowing consistently in the same direction, constantly reverses its direction, and changes its magnitude. Thus, let the curve (Fig. 74) be drawn with amperes as ordinates, and time as abscissæ. A continuous current will be represented by the straight line *a b*, an alternating current will be represented by the curved line *o c d e f*; that is, starting from zero, the alternating current gradually increases in value, until it reaches a maximum at *c*. It then decreases until at the end of the interval of time *o d* it has again reached zero. Its value then becomes negative, it begins to flow in the opposite direction, reaches a maximum at *e*, and again at the end of the interval of time represented by *o f*, passes through zero value. This process is repeated indefinitely. The curve representing the rise and fall of the current is usually assumed to be a sine curve, *i.e.*, the abscissæ being reckoned in angles, the sines of these angles give the values of the ordinates.

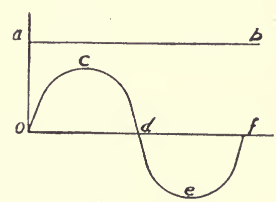


FIG. 74.

The sine curve is assumed because it is the simplest function, expressing a periodic change such as that of an alternating current. This assumption is also justified in practice by the fact that many alternators give a current curve which does not very materially differ from a true sine curve, and actual experimental facts, therefore, justify the adoption of a function which is necessary in order to make mathematical treatment of alternating-current phenomena possible. In problems where greater accuracy is needed, the mathematical fact that any periodic function whatever can be broken up into components, each of which is a true sine curve, can always be made use of. Going back to the simple assumption that an alternating current or an alternating E.M.F. may be represented by a sine curve, the number of times per second that a current goes through a complete cycle is called the periodicity of the current, and is generally represented by the symbol $\sim$. The periodicities in use in Great Britain at present are usually between 50 periods per second and 25 periods per second, represented by 50 $\sim$ and 25 $\sim$ respectively. A current of n periods per second is therefore one which goes through a complete cycle n times in one second and, if T represent the time taken for one complete cycle T, is evidently equal to $\frac{1}{n}$.

The current will have passed through a complete cycle and reached zero at the time T the sine curve has passed through a complete cycle, and reached zero when the angle is 360° or in circular measure 2π; the scale of time must therefore be such that $T = 2\pi$, and on such a scale any time t from the beginning of the cycle will correspond to the angle $\frac{2\pi t}{T}$. If C be the maximum value of the current, the value of the corresponding ordinate is $C \sin \frac{2\pi t}{T}$ and this, therefore, is the instantaneous value of the current at the time t. But $T = \frac{1}{n}$, and substituting this in the above expression, $C \sin 2\pi n t$ is obtained as the value of the current at the time t.

This is the final expression for instantaneous values of the current supposed to follow a sine law, and all that has been said as to current is of course equally true of E.M.F. curves. This expression is, however, somewhat lengthy, and it is not uncommon to simplify it by writing the symbol p instead of the expression $2\pi n$, the instantaneous value of the current is then C sin pt, and p has by some writers been called the pulsation of the current. This simplified form may always be used remembering that p is merely an abbreviation for $2\pi n$.

§ 2. Mean Value and R.M.S. Value of an Alternating Current.—In engineering formulæ, it is not usually the instantaneous value of the current or E.M.F. which is required to be dealt with; it is the average effect, and there are two values which are of importance. The mean value of the current over a half period can be found by plotting the current curve, finding its area by means of a planimeter, and dividing by the length of the base. The process is exactly similar to that of finding the mean pressure from a steam-engine indicator diagram, and it gives the mean value of the current. If this process be carried out on a sine curve it will be found that the mean value is ·636 times the maximum value.

By means of a simple integration it can easily be shown that the mean value of a sine curve is $\frac{2}{\pi}$ times the maximum value. If A be the maximum value $y = A \sin x$ will be the equation to the curve, and $\int_0^\pi A \sin x \, dx$ will be the area enclosed by the curve and the axis of x between values of $x = \pi$, and $x = 0$, and $\frac{1}{\pi} A \int_0^\pi \sin x \, dx$ will therefore be the mean value, but $\int_0^\pi \sin x \, dx = -\cos \pi + \cos 0 = 2$, ∴ the mean value of a sine curve, the maximum value of which is A, is $\frac{2}{\pi} \times A$ and $\frac{2}{\pi}$ is equal to ·636, nearly.

The heating effect, however, of a current, depends not upon the value of the current, but on the value of the square of the current; for instance, in a lamp of resistance ω, the watts absorbed are $= C^2 \omega$. If, therefore, it is desired to compare continuous and alternating values of current as regards their heating effect, it is the value of the current squared which must be considered. This can be done by plotting from the sine curve, another derived curve, the ordinates of which are obtained by squaring the sines. If the area of this curve be now taken, and divided by the base, the mean value of the current squared is obtained; taking the square root of this gives what is known as the root mean square current. This is usually denoted by the letters R.M.S. By actually plotting a curve and integrating it, it will be found that the value of this R.M.S. current is ·707 times the maximum value of the current. It is easily shown mathematically that it is $= \frac{1}{\sqrt{2}}$ times the maximum value, and this is approximately equal to ·707.

The same reasoning as used above for the sine curve shows the mean square value of a sine curve, the maximum value of which is A, to be

$$\frac{1}{\pi} A^2 \int_0^\pi \sin^2 x \, dx \text{ and } \int \sin^2 x \, dx = \frac{x - \sin x \cos x}{2},$$

∴ the mean square value $= \frac{1}{2} A^2$ and the square root of this is $\frac{1}{\sqrt{2}} A$.

The R.M.S. value of an alternating current is that which is most generally used. If, for instance, an alternating current of 20 amperes is mentioned without qualification, it must be taken to mean that the R.M.S. value of the current is 20 amperes, and for most practical purposes, this current is equivalent to 20 amperes continuous current. That is the reason why the root mean square value is that most commonly used in electrical engineering.

As a numerical example of the use of the constants determined above, consider the case of a current the R.M.S. value of which is 20 amperes. Such a current will in its heating effect be equivalent to a continuous current of 20 amperes. The maximum value will be $\frac{20}{\cdot 707} = 28\cdot 5$ amperes, the mean value will be $28\cdot 5 \times \cdot 636 = 18\cdot 2$ amperes, and the instantaneous value as the time t, the time t being reckoned from the beginning of a period, will be $28\cdot 5 \sin p\,t$ where $p = 2\,\pi\,n$ and n is the number of periods per second.

§ 3. Phase Difference and Addition of Currents Differing in Phase.—In Fig. 75 is shown a sine curve O A C D representing an alternating current, the maximum value of which is C. Suppose there be in another conductor a second alternating current of the same periodicity, but of maximum value of C_1. The

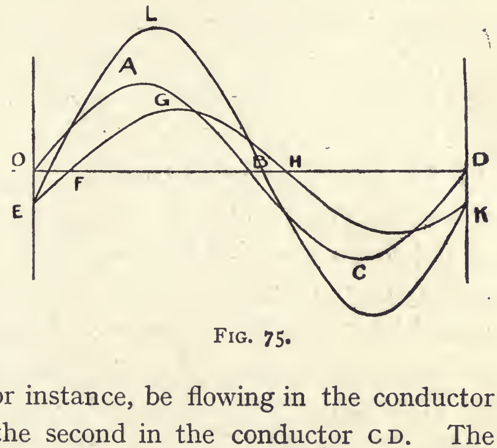


FIG. 75.

first current may, for instance, be flowing in the conductor A B (Fig. 76), and the second in the conductor C D. The current flowing in C D can also be represented by a sine curve, the maximum ordinate of which is $= c_1$, but in order completely to fix the relations between the two currents another point has to be considered. It is necessary also to notice what is known as the phase relation of the currents. The two currents may be such that they pass through zero value at the same time, or the times at which they pass through zero may be different.

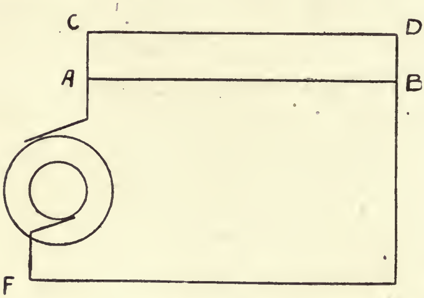


FIG. 76.

In Fig. 75 the second current is shown as lagging behind the first one by an interval shown by O F on the diagram. That is, the current in A B passes through zero value at a certain instant, and an interval of time represented by O F intervenes before the current in C D passes through its zero value. The two currents are then said to differ in phase and the length O F measures the difference of phase between them.

It is usual to measure difference of phase in angles, and according as O F represents 10, 20, 30, etc., degrees, so will the current in C D be said to lag behind the current in A B by 10, 20, 30, etc., degrees, the scale to which O F is measured in degrees being of course such that O D represents 360° (see § 2 above). The properties of the circuits which cause currents flowing in them to differ in phase will be discussed later. In Fig. 75 the second current is shown lagging 30° behind the first, O F having been chosen equal to 30°.

If now the two conductors A B and C D be connected together and form a third conductor B F, which has to carry both currents, what will be the value of the current in B F?

If continuous current were being dealt with it is well known that the current in BF would be the sum of the currents in AB and CD; the same is true of alternating currents, provided instantaneous values are taken.

At any instant the current in BF will be the sum of the currents in AB and CD, but it does not follow from this that the maximum current in BF will be the sum of the maximum currents in AB and CD. Such will only be the case if the maxima in the two branches occur at the same time, that is, if the two currents are in phase; if there is any difference of phase the maxima will occur at different times, and the maximum current in BF will be less than the combined maxima in AB and CD.

By adding together the ordinates of the sine curves OACD and EFGHK a third curve is obtained giving the instantaneous values of the current in BF. The curve so obtained ELBK is also a sine curve having the same periodicity as the other two and having a maximum value, say D, which is less than $C + C^1$.

The process can be carried out by trigonometry instead of geometrically. The instantaneous value of the current in AB will be $C \sin pt$ and the instantaneous value of the current, in CD $C_1 \sin (pt + 30)$, and by adding these together the required result will be obtained.

$$C \sin pt + C_1 \sin (pt + 30)$$
$$= C \sin pt + C_1 \cos 30^\circ \sin pt + C_1 \sin 30^\circ \cos pt$$
$$= \left(C + \frac{\sqrt{3}}{2} C_1\right) \sin pt + \tfrac{1}{2} C_1 \cos pt.$$

Let $M = C + \frac{\sqrt{3}}{2} C_1$ and $N = \frac{1}{2} C_1$,

then the above expression becomes

$$M \sin pt + N \cos pt.$$

Further, let ϕ be such an angle that $\frac{N}{M} = \tan \phi$

now $M \sin pt + N \cos pt = \sqrt{M^2 + N^2} \sin (pt + \phi)$,
for $\sqrt{M^2 + N^2} \sin (pt + \phi) = \sqrt{M^2 + N^2} (\sin pt \cos \phi + \cos pt \sin \phi)$ (I.).

But since $\frac{N}{M} = \tan \phi$,

$$\frac{N}{M} = \frac{\sin \phi}{\cos \phi} = \frac{\sin \phi}{\sqrt{1 - \sin^2 \phi}};$$

$$\therefore N^2 (1 - \sin^2 \phi) = M^2 \sin^2 \phi,$$

$$\therefore (M^2 + N^2) \sin^2 \phi = N^2,$$

$$\therefore \sin^2 \phi = \frac{N^2}{M^2 + N^2},$$

$$\therefore \sin \phi = \frac{N}{\sqrt{M^2 + N^2}}$$

In the same way it can be shown that $\cos \phi = \frac{M}{\sqrt{M^2 + N^2}}$.

Substituting these values in (I.)

$$\sqrt{M^2 + N^2} \sin (pt + \phi)$$

$$= \sqrt{M^2 + N^2} \left(\frac{M}{\sqrt{M^2 + N^2}} \sin pt + \frac{N}{\sqrt{M^2 + N^2}} \cos pt \right)$$

$$= M \sin pt + N \cos pt.$$

The value of the current in B F is therefore

$$\sqrt{M^2 + N^2} \sin (pt + \phi),$$

and substituting the values of M and N this becomes

$$\sqrt{\left(c + \frac{\sqrt{3}}{2} c_1\right)^2 + (\tfrac{1}{2} c_1)^2} \sin (pt + \phi)$$

$$= \sqrt{c^2 + c_1{}^2 + \sqrt{3}\, c\, c_1} \sin (pt + \phi) \text{ where } \tan \phi$$

$$= \frac{\frac{1}{2} c_1}{c + \frac{\sqrt{3}}{2} c_1} = \frac{c_1}{2c + 3\sqrt{c_1}}.$$

Which means that the current flowing in B F has a maximum value equal to $\sqrt{c^2 + c_1{}^2 + \sqrt{3}\, c\, c_1}$ and lags behind the current in A B by an angle ϕ, of which the tangent is $= \frac{c_1}{2c + \sqrt{3}\, c_1}$.

§ 4. **Clock Diagrams.**—The process of adding together currents or E.M.F.s which differ in phase can, however, be much simplified by dealing not with the instantaneous values, but with the maximum values only. If a straight line, say O A (Fig. 77) be imagined to revolve uniformly round the point O, so as to complete one revolution in the time T, it is evident that the projection of O A on the fixed line O E will represent the instantaneous values of a sine curve current completing one period in the time T and having a maximum value equal to O A. This follows from the fact that at any time t, measured from the beginning of rotation, the angle made by the line with its initial position will be $\frac{2\pi t}{T}$, or $2\pi n t$, where $n = \frac{1}{T}$, and the projection of the line on O E will at that time be O A $\sin 2\pi n t$, and this expression also represents the value at the time t of a current the maximum value of which is O A. Any other alternating current of the same periodicity can then be represented on the same diagram by some other straight line of a definite length from the point O, and making a definite angle with the line O A; this angle being equal to the difference of phase between the two currents.

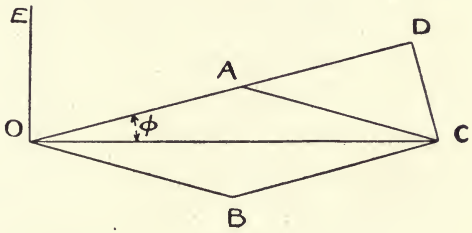


FIG. 77.

Problems involving alternating currents or E.M.F.s differing in phase are much simplified by representing each E.M.F. or current by a straight line on a diagram such as described above.

In the case of the two currents already dealt with, for instance, the straight line O A will represent in length C the maximum value of one current. OB must be drawn to represent in length C_1, the maximum value of the other current, and it must be drawn making an angle of 30° with OA, 30° being the difference of phase between the two currents. OA and OB are known as vectors, that is to say, not only their length is significant, but also their direction. The sum of two vectors is obtained by completing the parallelogram of which they form the sides, and drawing the diagonal through O, which then represents the resultant vector. A familiar example of this is the addition of forces by the parallelogram of forces. If this be done in Fig. 77, OC is obtained as the sum of the two currents, and it is useful exercise to compare the results of Fig. 77 and Fig. 75. By actually rotating the parallelogram round the point O, it will be readily seen that the projections of OA, OB, and OC, on the line OE, will, when plotted out, give respectively the curves OACD, EFGHK, and ELBK, and on the other hand by solving the triangle OAC in Fig. 77, the length of OC can be found as follows :—

$$OC^2 = OA^2 + AC^2 - 2\,OA, OC \cos OAC,$$

but OAC is equal to 180° − AOB, that is to 180° − 30°;

$\therefore \cos OAC = - \cos 30°,$

$\therefore OC^2 = OA^2 + AC^2 + 2\,OA\;OC \cos 30°$

and $\cos 30° = \frac{\sqrt{3}}{2}$.

$$\therefore OC^2 = OA^2 + AC^2 + \sqrt{3}\,OA\;OC,$$

$$\text{or } OC = \sqrt{OA^2 + AC^2 + \sqrt{3}\,OA\;OC},$$

and since OA represents C in magnitude, and AC represents C_1 this result is equivalent to that obtained by adding instantaneous values on page 168.

Again, producing OA to D and drawing CD perpendicular to OD, the angle DAC = 30°;

$$\therefore \text{CD} = \text{AC} \sin 30^\circ, \qquad \text{AD} = \text{AC} \cos 30^\circ,$$

and $$\tan\phi = \frac{\text{CD}}{\text{OA} + \text{AD}}$$

$$= \frac{\text{AC} \sin 30^\circ}{\text{OA} + \text{AC} \cos 30^\circ} = \frac{\frac{1}{2}\,\text{AC}}{\text{OA} + \frac{\sqrt{3}\,\text{AC}}{2}}$$

$$= \frac{\text{C}_1}{2\,\text{C} + \sqrt{3}\,\text{C}_1},$$

which again corresponds with the value for $\tan\phi$ found on page 168.

The representation of alternating currents or E.M.F.s in this way by vectors gives what is known as the clock diagram. On the clock diagram is indicated not only the magnitude of the currents dealt with, but also their phase relations, and in this way the most complicated problems may be solved. It should be noticed that the lines OA, OB and OC, for instance, strictly represent in the diagram maximum values only, but since the R.M.S. value or the mean value bears a constant ratio to the maximum value, the clock diagram may be taken to represent not maximum, but R.M.S. or mean value; it is simply a question of working to a different scale. E.M.F.'s can, of course, be dealt with in exactly the same way, and it is not unusual to draw clock diagrams in which both E.M.F.'s and currents are combined, and their phase relations to one another shown.

§ 5. **Self-Induction, Reactance and Impedance.**—As a simple instance of the use of clock diagrams, take a circuit of known resistance ω. If an alternating E.M.F. be applied to the terminals, the maximum value of which is E, and the number of periods per second n, it is required to find the current which will flow through the circuit. If continuous current were being dealt with, the current would be equal to $\frac{\text{E}}{\omega}$, but with alternating currents the results are modified by the well-known fact that any current gives rise to magnetic lines of force which surround the conductor. So long as the current is flowing steadily the number of these lines of force remains constant, and they do not influence the E.M.F. of the circuit, but when an alternating current is flowing through the conductors the magnitude of the magnetic field linked with it will be constantly varying in accordance with the variation of the current, and the cutting of the conductor by the lines of force as they alternately contract and expand, will give rise to an E.M.F.

Suppose L to represent the number of lines of force linked with the circuit at the instant when one ampere is flowing through, then L is known as the coefficient of self-induction or as the inductance of the circuit, and if the instantaneous value of the current is C sin $p\,t$, the number of lines at that instant linked with the circuit will be L C sin $p\,t$, and the E.M.F. generated on account of the cutting of these lines will be p L C cos $p\,t$.

The above implies that the number of lines of force is proportional to the current; this is strictly true only if there is no iron in the neighbourhood of the circuit, so that the whole of the magnetic path lies in air; if iron is present the number of lines of force, due to any current, is not proportional to the current, but it is nearly so for a considerable range of induction, and as the inductions at which alternating current apparatus is worked usually lie within this range, the assumption that the number of lines of force linked with the circuit varies directly as the current is sufficiently accurate for most purposes.

The E.M.F. of self-induction generated by the lines of magnetic force threading the circuit is evidently proportional to the rate of change in the number of these lines, and if this number varies directly as the current, it is easily seen that the maximum change occurs at the time when the current is zero. Examine a sine curve representing an alternating current, and therefore also representing the number of lines of force at any instant linked with the circuit, and it is seen that the most rapid change is occurring when the value of the current is zero, whilst the change becomes zero at those times when the current is at a maximum.

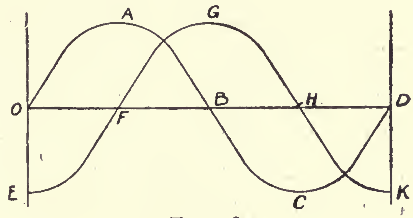


FIG. 78.

Thus if OABCD in Fig. 78 indicate the current, the E.M.F. of self-induction will be shown by a curve such as EFGHK. In the language of the differential calculus, if C sin pt expresses the current and LC sin pt the number of lines of magnetic force linked with it, $\frac{d(\text{LC}\sin pt)}{dt}$ will be the E.M.F. generated by the variations in the magnetic flux and $\frac{d(\text{LC}\sin pt)}{dt} = p\ \text{LC}\cos pt$. This indicates that the E.M.F. of self-induction will have a maximum value $= p\text{LC}$, and that it lags 90° behind the current C.

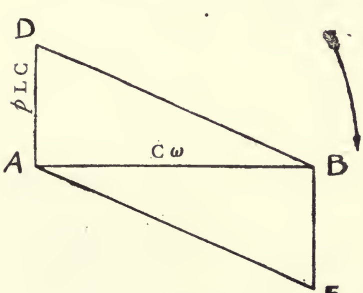


FIG. 79.

If, therefore (Fig. 79), a straight line AB be drawn to represent the E.M.F. absorbed in overcoming the resistance, the E.M.F. of self-induction will be repeated by AD. The length AB must be proportional to Cω, and the length AD to pLC. AD represents the E.M.F. actually generated in the conductor by the action of the current flowing through it. In order, then, that the current C should flow, it is necessary that at the ends of the conductor an E.M.F. be applied, represented by A E, and such that A E and A D will combine to give A B as their resultant, *i.e.*, A E must be parallel to D B.

Another way of looking at the matter which gives identical results is as follows: A B (see Fig. 80) again representing $C\omega$ and A D the E.M.F. of self-induction, it is necessary, in order to overcome this E.M.F. of self-induction, to apply an E.M.F. represented by A G, which shall be equal and opposite to A D; and therefore to a conductor of resistance ω, and inductance L, it is necessary, in order that a current C should flow, to apply an E.M.F. represented by A B, and also one represented by A G at right angles to it, therefore the total E.M.F. which must be applied at the ends of the conductor will be the resultant of A G and A B, that is A E. It is easily seen that A E is identical in Figs. 79 and 80, and from both these figures it is seen that if an alternating E.M.F. be applied to a circuit, having resistance and self-induction, the current will lag behind the applied E.M.F. by an angle θ such that $\tan\,\theta = \frac{p\,L}{\omega}$. In both figures the vectors are supposed to rotate clockwise as indicated by the arrow in Fig. 79. Or, again, taking the triangle A E B in which $AB = C\omega$, $BE = AD = pLC$, $AE = \sqrt{AB^2 + BE^2}$, *i.e.* the maximum value of the E.M.F. to be applied $=$ $\sqrt{C^2\omega^2 + p^2L^2C^2} = C\sqrt{\omega^2 + p^2L^2}$ or $C = \frac{E}{\sqrt{\omega^2 + p^2L^2}}$.

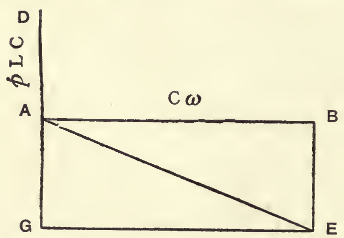


Fig. 80.

$\sqrt{\omega^2 + p^2L^2}$ is called the impedance of the circuit, and, as shown above, the impedance is the quantity by which the maximum E.M.F. must be divided in order to give the maximum value of the current. Or since the R.M.S. value is in either case obtained by multiplying the maximum value by ·707, it is equally true to say that the R.M.S. value of the E.M.F. divided by the impedance of the circuit gives the R.M.S. value of the current. Thus, just as in C.C. work the E.M.F. divided by the resistance gives the value of the current, so in A.C. work the E.M.F. divided by the impedance gives the current. But in addition it is to be noted that the current will lag behind the E.M.F. by an angle θ such that $\tan\ \theta = \frac{p\,L}{\omega}$.

Thus by drawing a right-angled triangle A B C (Fig. 81) such that A B is proportional to C ω when ω is the resistance of the circuit, and B C is proportioned to p L C, A B will represent the volts lost in ohmic resistance, B C the volts generated by the variations of the magnetic flux associated with the current, generally called the reactance volts, and A C the E.M.F. applied at the terminals of the circuit. Or if each of these terms be divided by C,

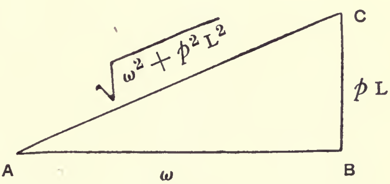


FIG. 81.

A B will represent ω the resistance,

B C will represent $pL = 2\,\pi\,n\,L$ the reactance,

A C will represent $\sqrt{\omega^2 + p^2\,L^2}$ the impedance of the circuit.

It is on these principles that the methods of finding the reactance voltage of the short-circuited coil of an armature, given in § 8 Chapter III., are founded; the inductance L is found on the assumption that one ampere-turn in the coil will give rise to 20 lines of magnetic force for each inch of core length, and L being found, it follows from the above that the reactance of the circuit is $2 \pi n$ L and the reactance voltage is $2 \pi n$ L C.

This is true only on the assumption that the current in the short-circuited coil follows during reversal a sine curve; this assumption is probably far removed from what actually happens, but it is made as being the only workable hypothesis, and it should be pointed out that the calculated reactance voltage is used as a rule only to compare new designs with others which have already been tried, so that it is the comparative value of the reactance voltage in different machines rather than its absolute value in any one case which is useful, and if all cases are worked out on the same assumption the comparative value of the results may be valuable even if the assumption is not very strictly justified.

§ 6. The Power Factor.—In continuous-current working, the power electrically given out by a generator or absorbed by any lamp or other apparatus is obtained quite simply by multiplying the volts at the terminals by the amperes; this gives watts which are the direct measure of the rate at which power is given out or absorbed. In the case of alternating currents, the result of multiplying R.M.S. volts by R.M.S. amperes gives the watts only in the special case when the E.M.F. and the current are in phase. Generally speaking, these will differ in phase by a certain amount, say by the angle θ. The angle θ is, as has been seen, called the angle of lag or of lead, and in order to obtain the watts a third factor must be introduced.

If E is the R.M.S. value of the E.M.F. at the terminals of a generator, for instance, and C is the R.M.S. value of the current, and the current differs in phase from the E.M.F. by an angle θ, then the watts given out by the generator are equal to E C $\cos \theta$. For this reason $\cos \theta$ is known as the power factor of the circuit. The product

E C, which in continuous current would be the true watts, is in alternating-current work known as the volt-amperes, and the rule is that, in order to obtain the watts, the volt-amperes must be multiplied by the power factor of the circuit. The power factor is shown to be equal to cos θ where θ is the angle of lag, as follows: In Fig. 82 let the sine curve O E B G D represent the E.M.F., say in a generator armature, and the curve K A F C H L the current flowing in the conductors, which lags behind the E.M.F. by an angle O A. If at any point the ordinates of these two curves be multiplied together, the result will be the watts generated at that instant, and if a number of such points be taken, and the results obtained plotted so as to give a third curve O A M B N C P D, this curve will represent the watts generated in the armature. It will not in general be a sine curve.

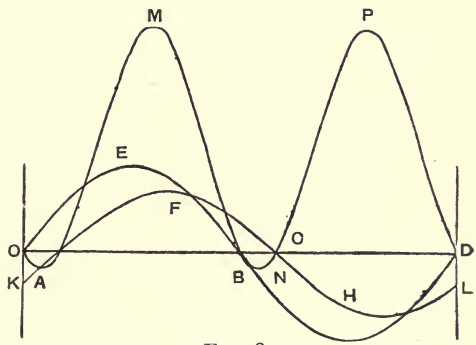


FIG. 82.

During the time O A the E.M.F. is positive and the current negative; the watts generated will, therefore, be negative during this interval—the generator will be absorbing power from the circuit, and not generating it. This is shown by the negative loop of the power curve. From A to B, both E.M.F. and current are positive, and the power curve shows a positive loop, but from B to C the E.M.F. and current are again of opposite signs, and the power curve is, therefore, below the axis. From C to D both E.M.F. and current are negative, and since two negative quantities multiplied together give a positive product, the loop C P D of the watts curves will be positive. The watts generated will, therefore, be constantly varying in value, and will sometimes be negative; the mean value of the watts generated will, however, be the area enclosed by the curve O A M B N C P D (reckoning areas above the line positive, and those below negative) divided by the length O D.

If E be the maximum value of the E.M.F., and C that of the current, the instantaneous values of E.M.F. will be $E \sin pt$, and of the current $C \sin (pt + \theta)$. Where O A represents the angle θ, if the curves are plotted to scale, and the power curve derived from them by taking the products of corresponding ordinates as ordinates be also plotted, the area of the latter curve can be taken by means of a plammiter, and the area divided by the length O D will be the mean watts generated. This will be found equal to $\frac{CE\cos\theta}{2} = \frac{C}{\sqrt{2}} \times \frac{E}{\sqrt{2}} \times \cos\theta$, and $\frac{C}{\sqrt{2}}, \frac{E}{\sqrt{2}}$ are the R.M.S. values of the current and E.M.F. respectively. This result is arrived at immediately by integrating; the area of the watts curve is $\int_0^{2\pi} E \sin pt \times C \sin (pt + \theta)\, dt$, and the mean value of the watts is accordingly $\frac{1}{2\pi}\int_0^{2\pi} E \sin pt \times C \sin (pt+\theta)\, dt$, which gives the above result $\frac{CE\cos\theta}{2}$.

It is evident that as the phase difference increases, the mean watts generated decrease, the area of the loops above the axis gradually diminishes, and the area of the negative loops below the line is increasing, until when the phase difference becomes 90°, the watts generated become zero. When the current and E.M.F. differ in phase by 90° they are said to be entirely out of phase.

Considered by means of a clock diagram identical results are obtained. In Fig. 83 the line O A represents the R.M.S. value of the E.M.F., and O B the R.M.S. value of the current lagging behind the E.M.F. by an angle θ. Draw A C perpendicular to O B, then the E.M.F. can be resolved into two components O D and O C, one of which, O D, is entirely out of phase with the current and will generate no watts, and one of which, O C, is in phase with the current, the watts due to which will be O C × O B, but O C = O A cos θ, and the watts generated will be O B × O A cos θ, and O A represents the R.M.S. value of the E.M.F., O B the R.M.S. value of the current; the result is therefore identical with that obtained above.

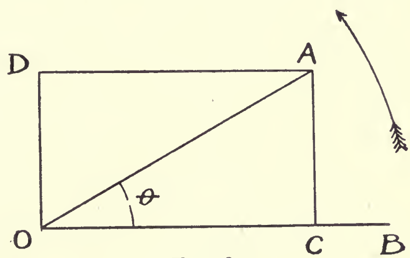


FIG. 83.

As an illustration of the use of these terms, suppose that it is required to design a generator to give 100 kilowatts at 500 volts. In continuous-current work this immediately settles the current. The current will be 200 amperes, because 200 amperes multiplied by 500 volts gives 100,000 watts. But if it is an alternating-current generator that is being designed, the current is not so easily determined. The amount of current which the generator must give out in order to give 100 kilowatts, and therefore to absorb the full power of the engine it is designed to work with, depends not on the machine itself, but on the power factor of the circuit it is to supply. If it is intended to do lighting only, the E.M.F. and current will be practically in phase, the power factor will be unity (when $\theta = 0°$, $\cos\theta = 1$), and 200 amperes will be required just as in the case of the continuous-current generator. If, however, part of the load is to consist of motors, the power factor will be less than one, and will depend on the size of the motors, and the load which they are carrying; it cannot be determined beforehand. It is therefore frequently specified that a generator shall be capable of absorbing the full power of the engine, on a circuit having a power factor of ·8, and if this is done in this case the maximum current which the generator must be capable of will be 250 amperes, because $500 \times 250 \times \cdot 8 = 100{,}000$ watts, which is the output the generator is required to be capable of.

§ 7. Polyphase Systems.—By combining E.M.F.s which differ from one another in phase are obtained the various systems known as polyphase systems. Of these, the systems chiefly in use in Great Britain are the two-phase, also known as the quarter-phase, and the three-phase.

In the first two E.M.F.s are produced differing in phase by 90°. In the three-phase system, three E.M.F.s are produced differing by 120°.

If three similar generators are coupled together rigidly in such a way that the armature coils of each come opposite the magnets not at the same instant, but in succession and at equal intervals of time, so that if a, b, c indicate the times in which the coils of the 1st, 2nd, and 3rd generator are opposite, say, a North pole, and a_1, the time when those of the first machine come opposite the next North pole, the intervals from a to b, b to c, and c to a_1 shall be equal, then from this combination of machines there will be obtained E.M.F.s which will differ in phase by 120°.

In practice this is done not with three different generators, but by fitting three sets of coils on one armature. Call these sets A, B, and C respectively, and place them on the armature so that they differ in position by 120 electrical degrees. By an electrical degree the following is meant. The angular distance from one pole to the next pole of the same polarity will, measured in actual degrees, depend on the number of poles on the machine, but whatever its distance in actual degrees, it is always to be reckoned as 360 electrical degrees. The spacing of the coils 120 electrical degrees apart therefore means that they equally divide the space between two poles of the same polarity. The coils of each of the sets A, B, and C are connected together in series, and led to two terminals. If the E.M.F. generated in each be assumed to be a sine curve, the relation of the E.M.F.s will be as shown in Fig. 84.

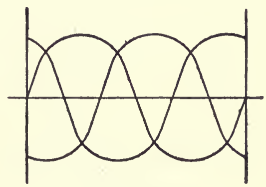


FIG. 84.

The three sets of coils being similar, the three E.M.F. curves will have the same maximum value, but they will differ in phase by 120°, as shown in the figure.

In practice, the three circuits of a three-phase machine are not usually kept independent. They are connected either as shown in Fig. 85, which is known as the "star" or Y connection, or as shown in Fig. 86, which is known as the triangle or delta connection. In either case three lines take away the currents from the three points marked *a*, *b*, and *c*. It might at first sight appear as if a fourth line would be necessary, in the case of the "star" connection, for the return of these currents to the generator. Inspection of Fig. 84, however, will show that the sum, at any instant, of the three currents is always

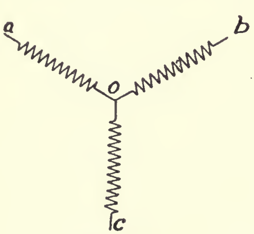


FIG. 85.

equal to zero, that is, that if the three currents are led, after passing through the lamps, the motor winding, or whatever other load is being supplied, to one common point, the sum of the currents flowing to that point will be zero, and therefore no fourth wire is required.

The advantages of using a polyphase rather than a single-phase system of distribution lie in the fact that polyphase induction motors are in every respect much more satisfactory than single-phase induction motors. There is also some saving in the weight of copper used in the line in the three-phase system. There has been a good deal of controversy as to the exact amount so saved. Copper in the transmission line can always be saved by working at a higher voltage, and evidently by combining the voltage of three different phases different results are obtained, according as the voltage between the different lines or the voltage between earth and line is considered. According as one or the other of these is taken as the standard of comparison, so different results are obtained as to the amount of copper required. The question, however, belongs to transmission of power rather than to dynamo design.

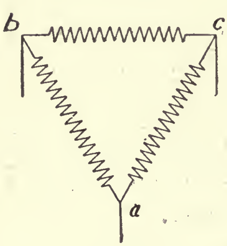


Fig. 86.

a, *b*, and *c* being the three terminals of the generator the difference of potential between *a b*, *b c*, and *c a* will be the same, and this is the voltage usually meant when speaking of the E.M.F. of a three-phase generator. If, for instance, it is required to design a three-phase generator to give 500 volts, this will always be taken to mean that the voltage between *a b*, *b c*, and *c a* must be 500 volts.

Usually the machine can be connected either star or delta; in either case the voltage must be the same, and the maximum current which it can supply to each line will be that specified as the output of the machine. The E.M.F. required to be generated in each winding will, however, differ according as the machine is connected star or delta. What relation the terminal voltage bears to the voltage generated in each winding can be shown by means of a clock diagram.

In Fig. 87 *o a*, *o b*, and *o c* represent the voltage generated in each winding. The angles *c o a*, *a o b*, and *b o c* are 120°, and the positive direction of the E.M.F. is in each case taken as from the centre point *o*. The E.M.F. between the terminals *a* and *b* will then be obtained geometrically by finding the resultant of *o a* and *o d*, where *o d* is opposite and equal to *o b*. This resultant is *o e*. From the construction it can be seen that *o e* is equal and parallel to *a b*—that is, if *o a* and *o b* represent the E.M.F. in the two phases, *a b* will represent the E.M.F. at the terminals.

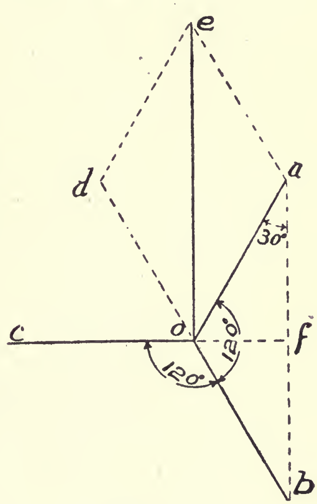


FIG. 87.

The reason for using *o d* and *o a*, not *o b* and *o a*, as the two components is worth a little further consideration. It illustrates one of the chief difficulties in the proper use of clock diagrams, which is to determine correctly the direction in which to represent E.M.F. or current. In the present instance it is seen that if the vectors *o a* and *o b* represented two independent E.M.F.s in the same circuit their resultant would be obtained by completing the parallelogram *b o a*, and would appear as a horizontal vector in the diagram. But in a three-phase generator there are two circuits connected at the point *o*, and the convention has been established that the direction from *o* to *a* and *o* to *b* shall be considered the positive direction in each respectively.

It is therefore obvious that in passing through the windings from terminal *a* to terminal *b*, the passage along one winding, *o b*, is in the positive and along the other, *a o*, in the negative direction, and that it is therefore not the sum, but the difference, of the two E.M.F.s which will be measured between *a* and *b*. One of the vectors must, therefore, be turned through 180° before completing the parallelogram. Which of the two shall be reversed in this way is a matter of indifference. If *o b* is reversed, as has been done in the diagram, it means that from *b* to *a* is reckoned the positive direction in measuring voltage at the terminals, and if *o a* had been reversed the resultant vector would have been of the same length, but in the opposite direction; from *a* to *b* would then have been the positive direction for measuring voltage. Whichever of these conventions is adopted it must be carried through consistently in dealing with the other phases, but is otherwise immaterial.

From the construction, the angles at *a* and *b* are each equal to 30°. Draw *o f* perpendicular to *a b*, then *a f* equals *o a* cos 30°, and *a b* is equal to 2 *a f*, therefore *a b* is equal to 2 *o a* cos 30° and cos 30° is equal to $\frac{\sqrt{3}}{2}$, therefore *a b* equals $\sqrt{3}$ *o a*, and this is for practical purposes sufficiently nearly equal to 1·73 *o a*. It may, therefore, be said that in the case of a star-connected generator the E.M.F. at the terminals is equal to 1·73 times the E.M.F. generated in each phase.

The current flowing in each line, however, must evidently be the same current as is flowing in the phase winding; one phase only is connected to each terminal, and there is no other winding supplying current to the line.

If a delta connection be used the cases are reversed. The E.M.F. between the terminals *a* and *b* is evidently equal to the E.M.F. generated in the winding *a b* (see Fig. 86), whilst the current flowing into the line at *b* is the vectors sum of the currents flowing in the winding *a b* and the winding *b c*. Exactly the same construction which has been used for the E.M.F. will show that this current is equal to 1·73 times the current in each phase.

To make this clear by a numerical example, suppose it is required to build a generator to give 500 volts at its terminals, and to supply 100 amperes per line. If this is wound "star," the winding of each phase will have to be suitable for giving $\frac{500}{1 \cdot 73} = 290$ volts, and will have to be of sufficient section to carry 100 amperes. If, on the other hand, the windings are connected "delta," each winding will have to have a sufficient number of turns to give 500 volts, but the section need only be capable of carrying $\frac{100}{1 \cdot 73} = 58$ amperes. The same arguments apply to the motors or to groups of lamps to be supplied from the generator. The windings of the motors or the groups of lamps may be connected either star or delta, and the same rules as to combining the currents or E.M.F.s will apply.

These statements are true, however, only when the system is balanced, that is to say, when the three phases are equally loaded and equal current is taken from each. If the whole load consists of motors, this condition is fulfilled or approximately so. The winding of each phase of the motors is supposed to be exactly similar, and any want of balance which may occur will be slight, and due to accidental differences in the process of manufacture. If lighting is also done by the generator, an attempt is generally made to balance the lamps by putting equal numbers on each phase, but the switching on or off of lamps will, to a certain extent, put the load out of balance. The calculations as to the combinations of currents and E.M.F.s when the system is not balanced become extremely complicated, and will not be dealt with here.

In a star-connected system the point at which the three phases meet is frequently referred to as the star point, and it is usually connected to earth, both at the generator and at the load end. This reduces the highest potential to which any point of the system can rise above earth in the proportion of 1 to 1·73.

The three-phase system is that most frequently used in Great Britain, and for this reason it has been dealt with first. Examples of two-phase are, however, to be met with, and the same rules must be complied with in combining the E.M.F.s and currents in such a system.

In a two-phase generator the coils are displaced by 90° instead of by 120°, and if the two sine curves of E.M.F. produced be drawn as in Fig. 88 it can be seen that, owing to a certain want of symmetry, the instantaneous sum of the two currents is not, as in the case of the three phases, always zero. In this case, therefore, a return wire cannot be dispensed with, but if one line be taken from each phase a third wire will be required returning to their common junction. The current which will have to be carried by this common line is equal to 1·41 times the current in each of the other lines. This is easily seen in the clock diagram in Fig. 89, where *o a* and *o b* represent the currents flowing in the two phases. At the junction of these phases they will combine to give a current represented by *a b*, and since *a o b* is a right angle *a b* is equal to $\sqrt{2}$ *o a* or *o b*.

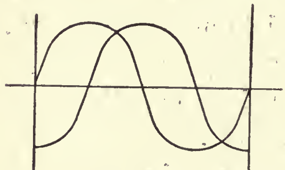


FIG. 88.

The $\sqrt{2}$ is equal to 1·41 nearly, and therefore the return wire will have to be nearly 50% greater than either of the others. It is usual, when working on a two-phase system, to keep the two phases entirely separate, and to use four lines, which are then all of equal capacity.

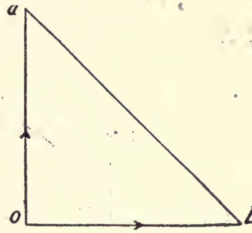


FIG. 89.

§ 8. Rotating Field Due to Polyphase Windings. — One very important property of polyphase windings is their power of giving rise to rotating magnetic field. This will be here illustrated by considering a three-phase winding, but the property is common to all polyphase systems. In Fig. 90 (1) three coils, A, B, and C, are indicated diagrammatically; they are arranged so that their planes are at angles of 120° apart, and three-phase current is led into the three coils. The three sine curves at the top of the figure represent the instantaneous values of the currents, and it is supposed that the current in coil A is represented by the curve A, the currents in coils B and C by curves B and C respectively. Consider the magnetic field produced at the instant shown by the line 1 in the figure.

This is shown by diagram 1 of Fig. 90. The current in A is at its maximum and will therefore cause a strong N pole at right angles to the plane of the coil; this is indicated by N; the currents in B and C are at half their maximum values, and will therefore give rise to weaker poles, also at right angles to their plane as indicated at *n* and *n*. The connections must be made so that the current flowing in B and C produces a field of the same polarity as A; if this condition is not observed the winding is not a true three-phase winding, and a uniformly rotating field will not be produced.

Diagrams 2 and 3 show the state of the magnetic field in the same way at the instants represented by 2 and 3 on the sine curves. In diagram 2 the current in C has reversed, and B is now at its maximum value. In diagram 3 A has reversed its direction, and C is at its maximum.

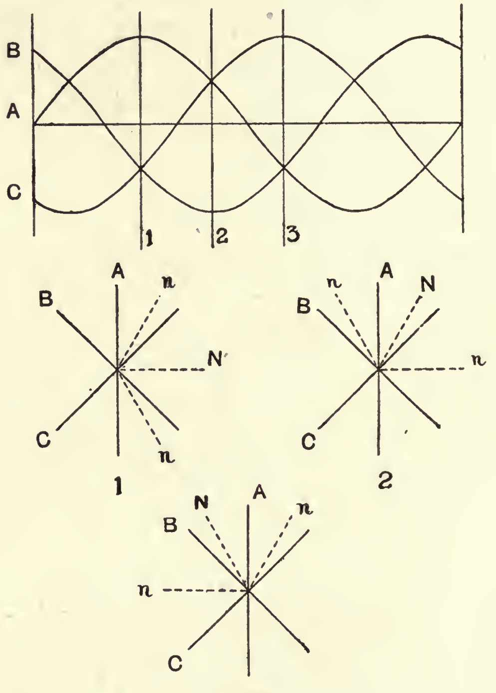


FIG. 90.

The effect on the field can be seen by inspection of the diagrams; in (1) the whole righthand side of the space occupied by the coils is of a north polarity, the field is strongest in the middle and weakest at the sides; in (2) this region of north polarity has moved towards the top of the figure, and in (3) is beginning to extend towards the left. There is thus produced a magnetic field rotating in a counter-clockwise direction.

Although for simplicity the north poles only have been indicated, there is of course a corresponding south pole directly opposite each. Of the fields created by the three coils, only the centre line has been indicated, but it is obvious that the fields will overlap and strengthen one another in the region between N and *n*, whilst outside this region the field is due to one of the coils only. As a matter of fact, the magnetic field, if not disturbed by the unequal distribution of iron parts of the circuit, will be represented by a sine curve having its maximum at N.

Obviously, when the currents have gone through one complete period, the state of matters will be exactly as shown in (1), that is, the sine wave of magnetic flux will have travelled once round the circle during the time occupied by one period of the current. The coils A, B, and C form a two-pole winding, but the same reasoning can be extended to any number of pairs of poles. If there are more than two poles, the field will not complete one revolution in the time of one period of the current, but will only pass through twice the pole pitch; *i.e.*, in the time that the current completes one period, the magnetic field rotates through 360 electrical degrees. By altering the connections to two of the coils so that the current shown by curve B flows in coil C and *vice versa*, the field will be made to rotate clockwise instead of counter-clockwise. This is easily seen by drawing the diagrams for positions 1, 2, and 3 over again with the altered relation of currents and coils. This fact is useful in practice, as it enables polyphase induction motors to be run in the reverse direction by merely changing the connections of their windings to the system of supply.

§ 9. Calculation of E.M.F. in an Alternator—Breadth Coefficient.—In calculating the E.M.F. generated in a C.C.

machine the formula $E \times 10^8 = Z P N \frac{R}{60}$ was used (see Chapter III.), where Z is the number of bars in series, P the number of poles, and R the revolutions per minute.

This formula is true, however, only of the mean value of the E.M.F. generated; different bars will have a different E.M.F. generated in them, according to their position in the field, but it is evident that since there are altogether P N lines of magnetic force entering the armature, and a bar makes one revolution in $\frac{60}{R}$ seconds, it will in that time have cut P N bars, and therefore its *average* rate of cutting lines must be $P N \frac{R}{60}$.

The effect of the commutator is to arrange that at any instant bars situated at all points of the armature are connected together so that their E.M.F.s add together in such a way that the E.M.F. at the brushes is obtained by multiplying the mean volts in each bar by the number of bars in series.

In the case of alternating-current machines, there is no commutator to take automatically the average of the E.M.F. generated at different points of the field, and it is therefore necessary to consider the rate at which conductors cut lines of force in such different positions.

Returning to the simple loop of wire rotated in a uniform two-pole field, made use of in Chapter II., in Fig. 95 let A B represent a section of the loop and imagine it turned through 90° so as to be in that position where it encloses the whole of the lines of magnetic force supposed to be uniformly distributed. Then, when the coil has moved through an angle θ the number of lines enclosed is obviously $\frac{OB}{OC} \times N$; but $\frac{OB}{OC} = \cos\theta$, and the number of lines is thus $N \cos\theta$.

If n is the number of revolutions per second, the coil will have passed through an angle 2π in $\frac{1}{n}$ second, or an angle $2\pi n$ in one second, and therefore the angle passed through in any time will be $2\pi nt$. Therefore at the time t the number of lines of force enclosed by the coil will be $N \cos 2\pi nt$, and as seen in § 5 of this chapter the rate of change of $N \cos 2\pi nt$ is $2\pi n N \sin 2\pi nt$. If the coil consists of T turns the E.M.F. generated will be $2\pi n T N \sin 2\pi nt$.

The following values are thus obtained—

$$E \text{ (max. value)} = 2\pi n T N = 6{\cdot}28\, n T N$$
$$E \text{ (R.M.S. value)} = {\cdot}707 \times 2\pi n T N = 4{\cdot}4\, n T N$$
$$E \text{ (mean value)} = {\cdot}636 \times 2\pi n T N = 4\, n T N.$$

In each case, if E is in volts it must be multiplied by 10^8 to bring it to C.G.S. units.

Note that the last formula, that for the mean value, corresponds to the formula for the E.M.F. of a continuous-current machine; for n being the number of revolutions per second in a two-pole field also represents the periodicity, and the periodicity divided by the number of pairs of poles, *i.e.*, by $\frac{P}{2}$ = revolutions per second.

$$\therefore n \times \frac{2}{P} = \frac{R}{60}. \qquad \therefore n = \frac{PR}{60 \times 2},$$

and, again, the number of turns, $T = \frac{1}{2}$ the number of bars,

$\therefore T = \frac{Z}{2}$ substituting in $E \times 10^8 = 4\, n T N$

$$E \times 10^8 = 4 \times \frac{PR}{60 \times 2} \times \frac{Z}{2} \times N,$$
$$= P \frac{R}{60} Z N,$$

which is identical with the formula given above for continuous current.

A further correction has to be made in the E.M.F. formula for alternators, because the different turns in the coil are not in identical positions in the field at the same instant. If it were practicable to put all the wires making up one coil in one slot in the case of a slotted armature, or all in one bundle at one spot of a smooth-cored armature, the E.M.F.s generated in individual wires would all be in phase and would add together, but if, as is generally the case, the wires are disposed over the armature so that they come into any region of the magnetic field in succession, the E.M.F.s generated in them will differ in phase, and the total E.M.F. measured at the terminals will be less than the sum of the separate E.M.F. generated in each wire.

If, for instance, a generator has 9 slots per pole, and E represents the maximum E.M.F. generated in the conductors in one slot, the distance from one pole to the next being 180 electrical degrees, each of the slots will be 20 electrical degrees removed from the next, and if E $\sin pt$ represent the E.M.F. generated in the middle slot, that generated in the neighbouring ones will be E $(\sin pt + 20°)$, E $\sin (pt - 20°)$, E $\sin (pt + 40°)$, etc. (see Fig. 91). The E.M.F. at the terminals will then be E $\sin pt$ + E $\sin (pt + 20)$ + E $\sin (pt + 40)$ + E $\sin (pt + 60)$ + E $\sin (pt + 80)$ + E $\sin (pt - 20)$ + E $\sin (pt - 40)$ + E $\sin (pt - 60)$ + E $\sin (pt - 80)$, and the

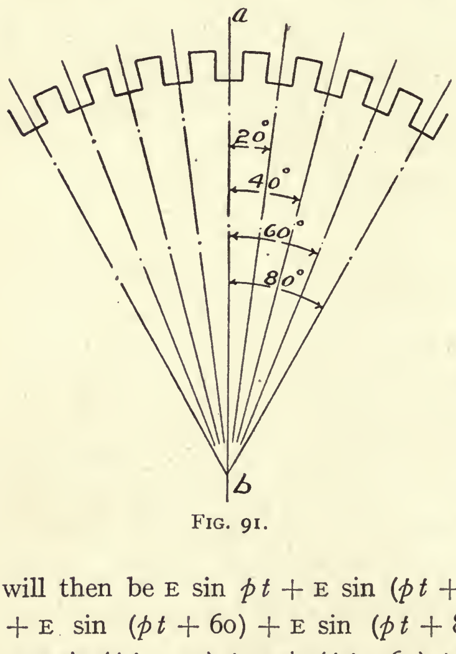


FIG. 91.

sum of all these terms is 5·786 sin $p\,t$, *i.e.*, the maximum value of the terminal E.M.F. will be 5·786 E. If the E.M.F. had all been in phase the maximum terminal E.M.F. would have been 9 E since there are 9 slots, and the value of the E.M.F. for one slot is E, and therefore the result obtained by multiplying the E.M.F. generated in each conductor by the total number of conductors will give a result which will be too large, and must be diminished in the proportion of 9 to 5·786. To get the actual E.M.F. generated, the E.M.F. calculated for the total number of bars in series, as would be done in C.C. work, must be multiplied by $\frac{5{\cdot}786}{9}$, which is = ·643.

This factor is called the breadth coefficient, as it depends on the breadth of the winding or on the angular space it occupies on the circumference of the armature. It is frequently denoted by the letter q.

The values of q for different cases are given in Parshall and Hobart's "Generator Design," worked out for different values of the pole pitch and for different breadths of coil.

For all practical purposes, it may be taken that the value of q varies from unity for a winding disposed in one slot per pole to ·636 for a distributed winding, *i.e.*, a winding in which the coils are spread over the whole of the armature face.

The formula for the E.M.F. (R.M.S. value) of an A.C. generator thus becomes

$$E \times 10^8 = 4{\cdot}4\, q \sim T\, N$$

where q is the breadth factor, $\sim$ the periodicity, T the number of turns in series, and N the number of lines per pole. In three-phase work, where the breadth occupied by one coil can never exceed one-third of the pole pitch, the value of q can never be very small; if a value $q = {\cdot}95$ be assumed, the formula becomes $E \times 10^8 = 4{\cdot}2 \sim T\, N$. The formula in this form has been used throughout Chapters VIII. and IX.

CHAPTER VIII

THREE-PHASE GENERATOR

§ 1. General Construction and Specification of an Alternator.—An alternating-current generator of the usual type consists of electromagnets excited by means of continuous current and of an armature carrying conductors in which an E.M.F. is induced. Since the current delivered to the external circuit is to be alternating, there is no need for a commutator, and the absence of the commutator allows of two important modifications in the general construction of the machine as compared with a C.C. generator.

In the latter the armature is invariably the rotating part, the magnets are stationary. This arrangement is necessitated by the fact that the brushes collecting current from the commutator must have a fixed position with respect to the magnets; if the latter revolved while the armature and commutator were stationary, it would be necessary that the brush gear should be carried by the magnets, and should revolve with them. Not only is this evidently unsatisfactory from a mechanical point of view, but it would also entail the addition of slip rings, and a second set of brushes to carry the current from the rotating brush gear to the stationary external circuit. This useless complication is always avoided in C.C. machines by making the armature revolve. In an A.C. generator, however, the armature may be the stationary part and the current led directly to the terminals without the need for any moving contacts; the continuous current required for exciting the magnets is led into two slip rings, to which the two ends of the magnet winding are connected.

There are certain distinct advantages in this method of construction : (1) It is easier to secure against the action of centrifugal force the comparatively small number of magnet coils than it is to secure in place a large number of armature coils disposed in many armature slots. (2) The voltage required from alternators is frequently fairly high ; 2,000 to 6,000 volts are quite common voltages. The E.M.F. of the exciting circuit can be kept as low as desired, and it is certainly preferable that the sliding contacts, the brushes and slip rings, should be in the low- rather than in the high-voltage circuit, especially in consideration of the fact that the exciting current even at a low E.M.F. does not usually become excessively high. (3) Two slip rings only are required for the exciting circuit ; in the case of polyphase generators, three or more rings would be required in the armature circuit if it were the rotating part. On these accounts the usual practice is to build alternators with internal revolving magnets, and a stationary external armature.

The second modification which may be introduced on account of the absence of a commutator is that the armature winding need not be symmetrical. Provided the right number of bars are connected in series, they may be connected in any order whatever. Symmetry is only required in a continuous-current armature because of the commutation troubles which would be caused by different coils being unlike in shape or position ; where there is no commutation to consider the shape of the different coils is immaterial.

Alternators may be built to give single-phase, two-phase, three-phase, etc., current.

From the considerations already noted in Chapter VII., three-phase current is that most generally used at present, and it is therefore proposed in this chapter to work out a three-phase generator with stationary external armature and rotating internal field magnets to give 300 kilowatts at 2,000 volts, and 25 periods per second when running at 375 revolutions per minute on a circuit, the power factor of which is ·8. The usual temperature rise of 70° F. above surrounding air will be specified, and in addition the specification will state the permissible variation of voltage under different conditions of load.

§ **2. Voltage Regulation.**—The regulation of an alternator depends not only on the machine itself, but also on the load which it is supplying, the regulation becoming much worse as the power factor of the load becomes smaller. This point will be further considered later. It will be assumed that the specification demands that the voltage shall not rise by more than 5% when the whole load is thrown off, if the load is non-inductive, and not more than 15% if the load has a power factor of ·8.

In considering the design of an alternator, heating and efficiency have to be considered in the same way as in a continuous-current machine, but there is no commutation, and many of the difficulties which have to be dealt with in C.C. machines are, therefore, not met with in the design of an alternator. On the other hand, the question of the inherent voltage regulation of an alternator is of much greater importance than in continuous-current machines.

The influence of the armature reaction becomes more serious when inductive loads are being supplied, and whilst these disturbances can be compensated for in C.C. generators by compound winding which will automatically compensate for changes of load, there is as yet no entirely satisfactory method of doing this in the case of an alternator. Many devices have been invented for automatically compounding an alternator, but they all involve considerable complications, and none has come into general practical use. Almost invariably the regulation of the E.M.F. is made by hand by introducing resistance either in the main circuit of the alternator field magnets or in the shunt circuit of the exciter. It is therefore of considerable importance that the inherent regulation of the generator should be as good as possible. Unfortunately improvement in the regulation is only obtained by increasing the size of the machine, and the amount of copper used on its magnets.

The number of poles on the magnets is fixed by the periodicity and the speed specified. Thus 375 revolutions per minute $= \frac{375}{60} = 6{\cdot}25$ revolutions per second, the periodicity specified is 25 per second, there must then be $\frac{25}{6{\cdot}25} = 4$ periods per revolution, and the number of periods per revolution is equal to the number of pairs of poles. The generator contemplated must therefore have 4 pairs of poles, that is 8 poles.

Next consider the current which will be required in each phase. The total output required is 300 kilowatts, or 100 kilowatts per phase. If the windings of the alternator are star connected, the E.M.F. in each will be $\frac{2{,}000}{1{\cdot}73} = 1{,}150$, and the current required $\frac{100{,}000}{1{,}150} = 87$. This is equivalent to 300 kilowatts so long as the current is in phase with the E.M.F., but if the load is inductive, say, consisting chiefly of motors, and is assumed to have a power factor of ·8, then the power given out to the circuit when 87 amperes per phase are being generated falls to $3\ (1{,}150 \times 87 \times {\cdot}8)$ $= 240{,}000$ watts, and in order to generate the full power specified the current must be increased to $\frac{87}{\cdot 8} = 108$. Each phase of the generator must therefore be capable of giving at its terminals an E.M.F. of 1,150 volts, and of carrying a current of 108 amperes. The volts lost through the resistance of the winding may be taken at about 2%, say 25 volts, bringing up the total of the E.M.F. to be generated to 1,175.

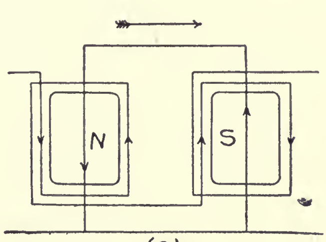
It would, of course, be possible to start the design by using a curve connecting D, the watts per revolution required from the machine, with $d^2 l$. Such a curve would differ from that used in Chapter III., as the dimensions of an alternator are found in practice to be greater for a given output than is the case for a C.C. generator. This curve is not, however, as useful a guide for alternating-current as it is for continuous-current generators, because the dimensions, the diameter and length of the armature, depend largely on the regulation required of the machine. For a more stringent specification as to regulation, a larger machine is required. For this reason and also as a useful example of how any design may be started from a different point of view, it is proposed to consider how the diameter and length of the armature may be arrived at tentatively from consideration of the voltage regulation required.

§ 3. Calculations of Armature Back Ampere-turns.— In considering the effect of load it is necessary to distinguish between the E.M.F. at the terminals of the machine, and the E.M.F. actually generated in the winding by its motion in a magnetic field. The former is called the terminal E.M.F., and the latter the impressed E.M.F.

The strength of the magnetic field present will depend on the number of ampere-turns on each magnet core, less any armature ampere-turns which may be opposing the magnet turns, and the impressed E.M.F. will be proportional to this field, and to the number of turns in series on the armature. The terminal E.M.F. will be equal to the impressed E.M.F. when there is no load on the machine, but as soon as current is taken from the alternator, the terminal E.M.F. will drop on account of the volts dropped in armature resistance; also since there is always an E.M.F. of self-induction generated in the armature winding, a component of the impressed E.M.F. will be required to overcome this, and not only will the terminal E.M.F. be less than the impressed E.M.F., but it will also differ in phase.

Even when the current is in phase with the terminal E.M.F. it will therefore not be in phase with the impressed E.M.F.; in order that it should be in the same phase it would be necessary that there be no inductance in the armature winding, which is of course an impossible condition.

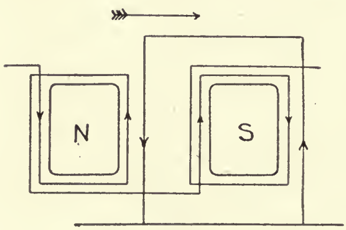
Thus even with a load having unity power factor the current will be out of phase with the impressed E.M.F., and much more so when the load has a low power factor, such as ·8. The fact that the current is out of phase causes a decrease in the E.M.F. much more serious than that due to the resistance of the winding, because a proportion of the armature ampere-turns then back against the magnet ampere-turns, and weaken the total flux in the air gap, and proportionately reduce the impressed E.M.F.



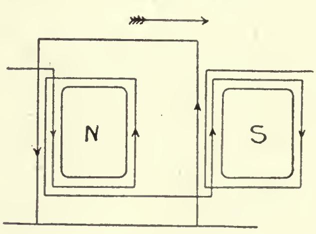
(a)
FIG. 92.

Inspection of Fig. 92 will show how the effect of the armature current is altered by its phase relation with the E.M.F. N and S represent two poles of the field magnets; the large rectangle shows an armature coil supposed to be moving over the poles in the direction of the arrow at the top of the figure. The coil is shown over the centre of the pole pieces, that is, at the time when the maximum E.M.F. is being generated in the armature winding. If the current is exactly in phase with the E.M.F. this will also be the instant of maximum current, but in the position shown the current in the armature coil is not backing against the magnet coils; it is producing a cross field, but gives no back ampere-turns.

Moreover, the arrangement is perfectly symmetrical; after a short interval, the conductor on the armature will have moved into the space between the magnet poles, and have got into the position shown in Fig. 93; in position "*b*" the current in the armature coil will have fallen below its maximum value, but, as indicated by the small arrows, what current is still flowing in the coils will be backing against the current in the magnet winding; this effect is, however, as shown in Fig. 94, entirely neutralised by the resultant of the armature current when in position "*c*." This position occurred before the coil arrived at the position "*a*," and it is seen that the direction of the current in position "*c*" is such as to increase the effect of the magnet winding. Thus so long as the current is in phase, with the E.M.F. for each position "*b*" of the coil on one side of the central position "*a*," there is a corresponding position on the



(*b*)
FIG. 93.



(*c*)
FIG. 94.

other side "*c*," where the current is of equal magnitude but produces the opposite effect on the magnet winding. These two effects therefore neutralise one another, and there are no armature back ampere-turns.

If, however, the current lags behind the E.M.F., this symmetry is destroyed. The maximum value of the current occurs not when the coil is in position "*a*," but some time later, and therefore for every position such as "*b*," the current will be stronger than it is in the corresponding position "*c*," and in the position "*b*" it is backing against the magnet coils; the total effect, therefore, will be a weakening of the total number of magnetic lines, and therefore a decrease in the E.M.F. generated.

The magnitude of this effect depends upon the angle of lag; if ϕ be the angle of lag so that $\cos \phi$ is the power factor, then the total number of armature ampere-turns per pole $\times \sin \phi$ is the number of back ampere-turns acting in opposition to the magnet winding. This is most easily seen by considering the effect of an armature coil in a two-pole field (see Fig. 95). If the current reaches its maximum value at the time when the coil is in the position O C then B C is proportional to the component of the ampere-turns which act in opposition to the magnet ampere-turns, and $BC = OC \sin \phi$. If $\phi =$ zero, that is if the current is in phase with the impressed E.M.F., then $\sin \phi = 0$ and there are no back ampere-turns; this agrees with the statement above deduced from Figs. 92, 93 and 94.

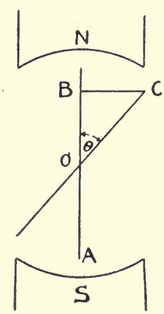


FIG. 95.

The number of ampere-turns on the armature effective as back ampere-turns will thus depend (1) on the power factor of the circuit, (2) on the reactance of the armature winding. If the power factor of the load is unity, the external current is in phase with the terminal voltage, but since there must necessarily be a reactance effect in the armature itself, it will not be in phase with the impressed E.M.F., and will thus cause a weakening of the main flux. When the power factor of the load is nearly unity, the effect due to the reactance of the armature is of importance, but if the power factor of the external circuit falls to say ·8 or ·9, the reactance voltage of the armature circuit becomes of relatively small importance.

In Fig. 96 let O A represent the E.M.F. required at the terminals of the generator, plus the volts dropped in the resistance of the winding. If the power factor of the outer circuit be unity, the vector O A will also represent in direction the external current, since this will be in phase with the terminal E.M.F., and if O B represent the reactance voltage of the armature, O C will represent the E.M.F. necessary to give, when combined with the reactance voltage, the required terminal E.M.F. together with the volts dropped in the winding; that is, O C represents the required impressed E.M.F., and the angle A O C, denoted by ϕ in the figure, represents the angle of lag of the current behind the impressed E.M.F. It is evident that, since B A is equal and parallel to O C, B A may be taken to represent the impressed E.M.F. in magnitude and direction, and that the angle B A O represents ϕ; it is, therefore, not necessary to draw the whole parallelogram,

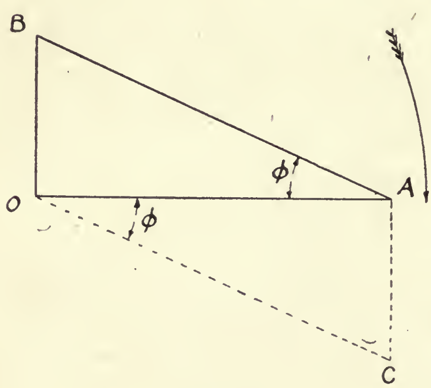


FIG. 96.

the triangle A O B is sufficient, and in clock diagrams which tend to become complicated, it is frequently usual to leave out half of the complete parallelogram in order to avoid confusion from too large a number of lines.

The modifications required when the P.F. of the outer circuit is not unity are shown in Fig. 97. In this figure O A is proportional to the terminal voltage + volts dropped in the ohmic resistance of the armature; O B represents again the reactance of the armature winding. The current now lags behind the terminal E.M.F. by an angle θ such that $\cos \theta =$ power factor of the circuit. Draw O C, making an angle θ with O A, and make O C = O A, then B C represents in magnitude and direction the impressed E.M.F. required; and if C D be drawn parallel to O A the angle D C B represents the angle of lag of the current behind the impressed E.M.F.

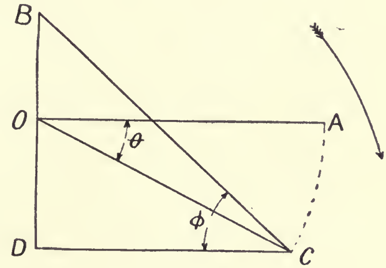


FIG. 97.

In the generator dealt with in this chapter the regulation is specified with a power factor of ·8, $\therefore \cos \theta = \cdot 8$. Reference to a table of trigonometrical functions will show that if $\cos \theta = \cdot 8$ $\sin \theta = \cdot 6$, and in the triangle O D C, O C D $= \theta$; $\therefore$ O D = O C $\sin \theta$ and D C = O C $\cos \theta$. Again $\tan \phi = \frac{\text{BD}}{\text{DC}}$ $= \frac{\text{BO}+\text{OD}}{\text{DC}} = \frac{\text{BO}+\text{OC} \sin \theta}{\text{OC} \cos \theta}$, and it has been shown that for a P.F. of ·8

$$\sin \theta = \cdot 6 \quad \cos \theta = \cdot 8. \qquad \therefore \tan \phi = \frac{\text{BO}+\cdot 6 \text{ OC}}{\cdot 8 \text{ OC}}.$$

O C is equal to 1,175 volts (see page 198). O B cannot be calculated till the armature winding has been settled, but previous experience on machines of this size shows it to be small compared to O C, say about 120 voltsf

$$\therefore \tan \phi = \frac{120 + \cdot 6 \times 1{,}175}{\cdot 8 \times 1{,}175} = \cdot 82.$$

Reference to a trigonometrical table shows that if tan θ $= \cdot 88$ sin $\theta = \cdot 66$, the armature back ampere-turns when the P.F. is ·8 will be ·66 × the total armature ampere-turns per pole.

In order to ensure that the voltage shall not rise more than 15%, when full load is thrown off, it is advisable that the back ampere-turns should not be more than 20% of the ampere-turns on one magnet pole. It will be possible in the present instance to put about 11,000 ampere-turns on each magnet core, and the armature back ampere-turns may, therefore, be $\frac{11{,}000 \times 20}{100} = 2{,}200$. Since the back ampere-turns are ·66 of the total armature ampere-turns per pole, the latter may amount to $\frac{2{,}200}{\cdot 66} = 3{,}300$.

The reason why a difference of as much as 20% in the number of ampere-turns may be expected to make not more than 15% difference in the E.M.F. generated is, that if care be taken to work the iron parts of the magnetic circuit at fairly high densities, the magnetisation curve of the machine will be flat, and a given percentage change in the ampere-turns will make a much smaller proportional change in the total flux. This will be reverted to later in § 9, when sufficient data have been obtained to enable the reactance voltage of the armature to be calculated, and the magnetisation curve of the machine to be drawn.

§ 4. **Armature Dimensions and Number of Bars.**— Let it then be determined that the armature of the generator should not have more than about 3,300 ampere-turns per pole, the R.M.S. of the current per phase is 108, and the maximum value, therefore, $\frac{108}{\cdot 707} = 154$. Multiply by two to get the effect of the three phases, this gives 308, and $\frac{3{,}300}{308} = 10{\cdot}8$; say, ten turns per pole per phase.

Multiplying the effect of one phase by two in order to get the effect of the three phases, is based on the consideration that when the current in one phase is at its maximum, the currents in the other two phases are equal and have half their maximum value (see Fig. 84). Taking the effect of one phase as 1, the effect of the three is therefore $1+\frac{1}{2}+\frac{1}{2}=2$. Since this relation between the currents is only instantaneous and varies at different parts of the sine curve, this method is not strictly correct; it is justified by giving results which are found in practice to be sufficiently accurate.

Ten turns per pole per phase will mean, since these are 8 poles, $10 \times 8 = 80$ turns per phase, and altogether $80 \times 3 \times 2 = 480$ total bars on the armature. The E.M.F. per phase is 1,175 volts, and substituting in the E.M.F. formula (Chapter VII.) the value of the flux per pole required, $N = \frac{1{,}175 \times 10^8}{4{\cdot}4 \times q \times 80 \times 25}$ assuming for q, a value of $\cdot 95$, this gives $N = 14 \times 10^6$.

Working at a density B in the air gap of 8,000 will require an air gap area of $\frac{14 \times 10}{8{,}000} = 1{,}750$ square centimeters $= \frac{1{,}750}{6{\cdot}45} = 273$ square inches. This area can be obtained by taking a length of core $= 20''$ and a pole arc $= 13{\cdot}6''$. Say that the pole arc is equal to $\cdot 7$ of the pole pitch, then the pole pitch $= \frac{13{\cdot}6}{\cdot 7} = 19\frac{1}{2}''$, and there are 8 poles, therefore, the circumference $= 19\frac{1}{2} \times 8 = 156''$, and the diameter $= \frac{156}{\pi} = 49\frac{1}{2}''$.

The diameter and length of armature core are thus arrived at as 49½″ and 20″ respectively. In this instance, since the armature is to be external to the field magnets, 49½″ is its inside diameter. The slots carrying the winding will be on the inside periphery of the core, the depth of slot can, therefore, be increased to any reasonable amount without unduly increasing the tooth densities at the root. This is due to the fact that with slots on the internal periphery the tooth gets broader at the root instead of narrower, as is the case in an armature having external slots. It is, in fact, not space on the armature which is the limiting factor, but space for the magnets which are inside the armature. On this account it is well to reverse the order followed in Chapter III., and to calculate first whether 49½″ is a sufficient diameter to allow of getting in the field magnets.

§ 5. **Magnet Windings.**—Each magnet core has to carry 14×10^6 lines of force plus the leakage lines which do not enter the armature. The magnets being more crowded together by being the internal member of the machine, the surfaces of different poles will be closer together and the leakage paths therefore shortened. The leakage will be greater than in the case of external magnets, and it will be well to allow a leakage coefficient of 1·25. The total number of lines to be carried by each magnet core will then be $14 \times 10^6 \times 1{\cdot}25 = 17{\cdot}5 \times 10^6$. Working at a density of about 15,000 lines per square centimeter will require an area of, say, $\frac{17{\cdot}5 \times 10^6}{15{,}000} = 1{,}160$ square centimeters $= \frac{1{,}160}{6{\cdot}45} = 180$ square inches, and since the length of core parallel to the shaft is 20″, the width must then be $\frac{180}{20} = 9''$ (see Fig. 98).

The radial length of the magnet core including air gap and pole shoe will have to be about 9″ to allow of sufficient length of winding space; the diameter at the bottom of

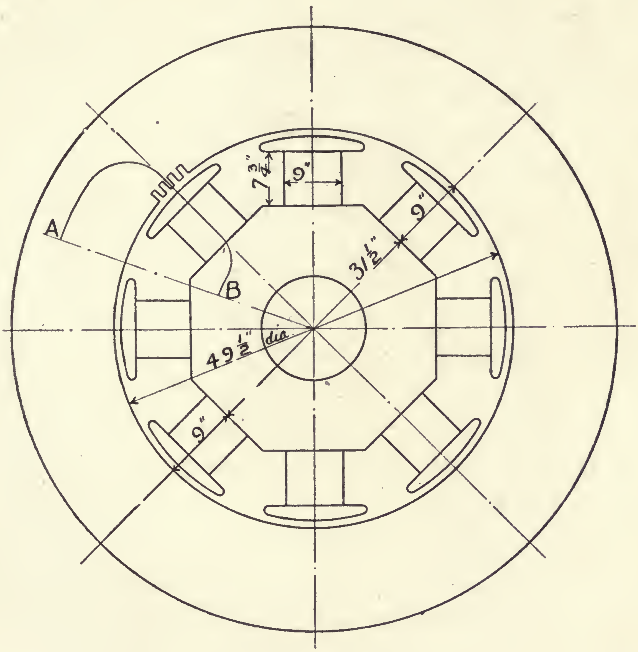


FIG. 98.

the magnet core will thus be $49\frac{1}{2}'' - (9 \times 2) = 31\frac{1}{2}''$, and the circumference will be $31\frac{1}{2} \times \pi = 98\frac{1}{2}''$, this divided by 8, the number of poles, give 12·4″, and the core itself is 9″, leaving 3·4″ between the magnet cores to accommodate the winding.

Each magnet coil must have at least 11,000 ampere-turns in order that the ratio of armature back ampere-turns to magnet ampere-turns may not be excessive. The winding necessary to give these will depend upon the voltage of the exciter.

The exciter is in many cases supplied with the alternator, and it is not used for any other purpose than exciting the magnets. In central stations, however, separate continuous current dynamos are frequently provided which supply the magnetising current for all the alternators in the station, and are sometimes used for other purposes also, such as station lighting or supplying motors driving auxiliary machinery. In the former case the voltage of excitation can be chosen to suit the special alternator being designed, in the latter the alternator magnet winding must be suitable for the voltage of the exciting sets already installed.

In either case a low voltage is advisable, because the lower the voltage the fewer turns of conductor are required on the magnets. An increased current is of course necessary at the low voltage and there is no saving of copper in the magnet winding whatever the voltage chosen, but a few turns of copper of a large section are more easily wound than a larger number of smaller section, and less space is wasted in insulation.

A frequent voltage to use for excitation purposes is 100 volts, and many of the recent alternators, direct coupled to turbines and carrying the exciter on the same shaft, are excited at as low a voltage as 60 volts.

On the assumption that 100 volts are chosen for the exciter it will be necessary to put on each coil such a winding that 11,000 ampere-turns per pole will be obtained. The E.M.F. on each coil will be $\frac{100}{8} = 12{\cdot}5$, in order to allow a margin for contingencies, say 10 volts per coil. The cooling surface for each coil can be found in the same way as in Chapters III. and IV. Assuming the radial length of the winding space to be 8″, and the mean depth of winding 2″, the mean turn will be $2\ (20\frac{1}{2} + 9\frac{1}{2}) + 2\pi = 66\frac{1}{2}$ and the cooling surface $\{2\ (20\frac{1}{2} + 9\frac{1}{2}) + 2\,\pi \times 2\} \times 8 = 580$ square inches (see Fig. 99).

Since the magnets are rotating, the cooling effect will be greater than in the case of stationary magnets, and it is safe to allow about ·8 to ·9 watt per square inch of surface. Taking ·9 watt to the square inch, the watts which can be radiated will be $\cdot 9 \times 580 = 520$ per coil, and since the E.M.F. on each coil is 10 volts, the current must be $\frac{520}{10} = 52$. The number of turns required to give 11,000 ampere-turns will be $\frac{11,000}{52} = 210$, and the resistance of one coil must be $\frac{10}{52} = \cdot 192$ ohm.

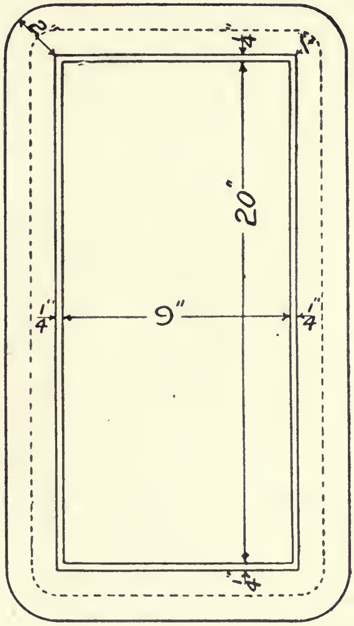


FIG. 99.

Again, since the mean turn is $66\frac{1}{2}''$ the length of winding will be $205 \times \frac{66\frac{1}{2}}{21} = 1,140$ ft., and the resistance per foot must thus be $\frac{\cdot 192}{1,140} = \cdot 000168$ ohm.

A copper conductor having a section of ·065 square inch has a resistance of ·00017 ohm per foot. This may conveniently be wound on the magnets in the shape of a wide thin strip having, say, a width of $3\frac{3}{4}''$, and a thickness of $\cdot 017''$. This can be wound by folding the strip in the middle of its length as shown in Fig. 100, and winding the two ends in opposite directions; say that the end *a* is wound right-handed, sixty-two turns, one on the top of the other, being taken, and similarly 148 turns of the end *b* wound lefthanded, a coil will thus be obtained having 210 turns, and of the shape shown in Fig. 101.

The advantage of this method of winding is that the two ends of the strip are on the outside of the coil, and

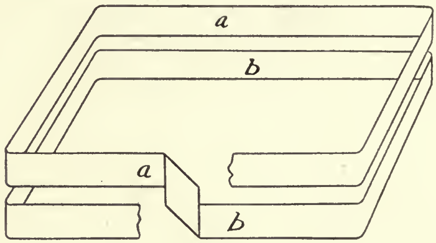


FIG. 100.

available for taking off connections without any waste of space in fetching out a connection from an inside end. The insulation may consist of strips of cotton cloth say 4″ wide, wound on at the same time as the copper strip so as to insulate each turn from the next. The whole coil after winding is saturated in some good insulating varnish, and baked at a temperature of about 180° F., which makes it quite solid. The two strips side by side will take up a radial length of the core of $2 \times 3\frac{3}{4} = 7\frac{1}{2}$, allowing $\frac{1}{4}''$ between the two turns, and $\frac{1}{8}''$ at each end, the total radial length of the winding will be 8″. The depth of winding, allowing ·01″ for the thickness of insulation between the turns will be $62 \times (\cdot 017 + \cdot 01) = 1\cdot 55''$ for the bottom half and $148 \times (\cdot 017 + \cdot 01) = 3\cdot 7''$ for the top half, the coil will therefore go into the space available on the magnet cores. Two neighbouring coils are shown in position on

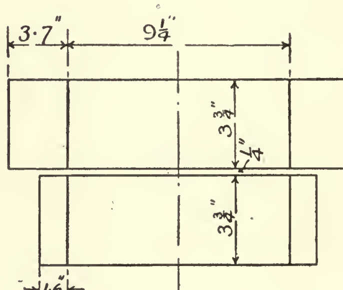


FIG. 101.

the magnet cores in Fig. 102. The space between two magnet cores narrows rapidly towards the centre, and it is to adapt the coil to this that the bottom half of the coil has only 62 turns while the top half has 148.

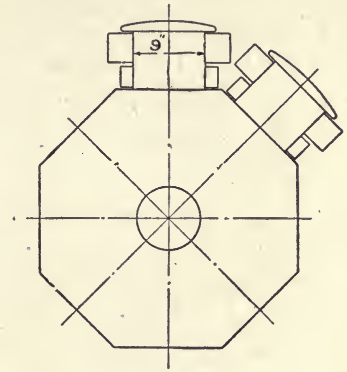


FIG. 102.

Strips of wood should be placed in the ¼" space between the turns, and also at the ends to prevent any possibility of the bare edges of the copper coming into contact with each other or with the metal of the magnet.

§ 6. Armature Winding.—The armature conductors will be 480 in number, and the section must be sufficient to carry 108 amperes (see page 205). At 2,000 amperes per square inch, this will require a section of about ·05 square inch, say a bar ·3" × ·175". These may be disposed two bars per slot in 240 slots. The allowances for insulation and the necessary size of slot can be calculated in the same way as already exemplified in Chapters III. and IV.; the insulation in the present instance must be sufficient to withstand a pressure of 2,000 volts.

When the E.M.F. exceeds 500 volts it is usual to employ mica for the slot insulation. For low voltages the thickness of insulation between the conductor and the side of the slot is determined merely so as to give the necessary mechanical strength, but for higher voltages, a certain minimum thickness is required to prevent breaking down and flashing through to the iron of the armature core.

It is also specially necessary to take precautions where the bars come out of the slot that the insulation be not injured by bending. The insulating trough should project so far out of the slot as to give a considerable length of leakage path from copper to iron. For a 2,000-volt machine the bars should come straight out of the slots for at least $\frac{1}{2}''$ before being bent, the mica trough lining the slots will project by that amount at each end. Allowing ·035″ for taping the bars, and a thickness of ·05″ on each side for the mica trough, the total depth and width of slot required will be :—

	Depth	Width
Copper and tape .	2(·3 + ·035) = ·67″	·175 + ·035 = ·21″
Mica . . .	·1″	·1
Wooden wedge .	·187″	—
Slack . . .	·043″	·04″
	1·000″	·35″

Each slot must therefore be 1″ by ·35″.

If this be tested to see that there is room on the armature for this number of slots, it will be found that the pitch of the slot is $\frac{49 \cdot 5}{240} \pi = \cdot 65$, and each tooth at the air gap (where it is narrowest) will have a width of ·65″ − ·35″ = ·3″. There are under each pole $\frac{240}{8} \times \cdot 7 = 21$ teeth, and allowing 10% for fringing, say 23 teeth, to carry the flux from one pole. In order to find the amount of iron available it is necessary to know the effective length of the armature core.

The total length is 20″, and from this must be deducted the amount taken up by ventilating ducts and by insulation between the discs. Suppose 6 ventilating ducts are provided, each half an inch wide, and that 10% is taken off for the insulation between discs, then the effective length will be (20 − 3) × ·9 = 15·3″. The amount of metal in the teeth carrying the lines from one pole will be $23 \times 15{\cdot}3 \times {\cdot}3 = 105''$, and $105 \times 6{\cdot}45 = 680$ square centimeters; thus the value of B (the magnetic induction at the narrowest part of the teeth), is found to be $\frac{14,000,000}{680} = 20,500$.

This value is less than the inductions at the bottom of the teeth in the C.C. machines, and might be increased without disadvantage. It must, however, be noticed that whilst a high tooth density helps the commutation in C.C. machines, there is no particular reason why very high inductions should be employed in an alternator, and although it might at first sight seem that the diameter of the machine could be reduced by lessening the tooth pitch, on further consideration it will appear that the diameter cannot be reduced without unduly restricting the space available for the magnet windings. In the case of internal magnets it is the winding space required for the magnets which settles the smallest diameter that can be used, and just as in a C.C. machine with internal armature the minimum diameter is fixed by the space required for slots and teeth, and the magnets, the external member, have generally more space available for winding than is actually required, so it is not astonishing to find that when the armature becomes the external member there is more room available on it than is actually required.

The connections of the different bars forming one phase, which have to be all connected in series, might be arranged in exactly the same way as in a C.C. armature, but as there is no need to preserve any symmetry in the winding they may more conveniently be arranged in rectangular coils, each of these may have twenty turns, and will include the bars, contained in ten slots under one pole and ten slots under the neighbouring pole. There will be four such coils in a phase since there are four pairs of poles, and in the three phases, there will be twelve coils.

Phase A	Phase B	Phase C
1 to 10	11 to 20	21 to 30
31 to 40	41 to 50	51 to 60
61 to 70	71 to 80	81 to 90
91 to 100	101 to 110	111 to 120
121 to 130	131 to 140	141 to 150
151 to 160	161 to 170	171 to 180
181 to 190	191 to 200	201 to 210
211 to 220	221 to 230	231 to 240

If one phase be called phase A, another B, and the third C, the table above shows how the slots will be divided between the three phases. The bars in slots 1 to 10 will connect to bars in 31 to 40 to form one coil, and so on. Fig. 103 shows the arrangement of the end connections; alternate coils, as they come out of the slots are bent downwards, so as to pass under the ends of the coil belonging to another phase which has to cross them. Inspection of this figure will show that this arrangement is adequate, provided the coils are properly arranged. If bars in slots 11 to 20 in phase B were connected to bars in 41 to 50, an arrangement which would be electrically quite suitable, they would have to cross bars both of phase A and of phase C, which already occupy the two different planes of end connections, and a third plane would have to be provided, the coil of phase C would have to bend still farther downwards, and space would thus be uselessly

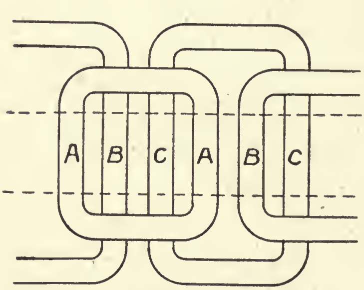


Fig. 103.

taken up. By taking the connections of one phase in the opposite direction round the armature, the necessity for three rows of connections is avoided; this is the reason why in phase B the bars in slots 11 to 20 connect to bars in slots 221 to 230, instead of to the bars in 41 to 50.

The four coils composing one phase are connected in series, and of the two ends of each phase one is brought to a terminal, the other connects with the other two phases forming the star point. To ensure that the phases are connected up in the correct direction, it is only necessary to notice that the ends connected together to form the star come out of slots which are 120 electrical degrees, not 60 electrical degrees apart; thus ends from slots 1, 21, and 41 may be connected together to form the star point.

§ 7. Armature and Magnet Losses — Heating and Efficiency.—The calculation of the losses in the armature and magnets, and the calculations of the probable temperature-rise and efficiency present no new features in the case of an alternator, and can therefore be briefly dealt with. Consider first the losses in the armature winding at full load. The winding consists of 10 turns per pole per phase and each phase will therefore have 80 turns, the section of the conductor is $\cdot 3 \times \cdot 175 = \cdot 525$ square inch. The mean length of one turn can be seen from Fig. 104, the pole pitch is $19\frac{1}{2}''$, and the length of conductor in the slot is $20''$, allowing on the average $4''$, as shown in the figure, for the length projecting beyond the core and for the necessary bends; the total length of one coil works out at $2 \times (20 + 19\frac{1}{2} + 8) = 95''$. The

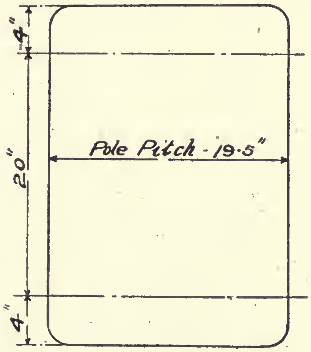


FIG. 104.

total length of conductor in one phase will therefore be $\frac{95}{12} \times 80 = 630$ ft., and the resistance of 630 ft. of copper conductor ·0525 square inch in section will be found from the copper table in the Appendix to be ·112 ohm. The watts lost in each phase when 108 amperes per phase are being taken from the generator will therefore be $(108)^2 \times \cdot 112 = 1{,}320$ watts, and the total copper loss for the three phases 3,960 watts.

The iron losses in the armature core will be found in the same way as in Chapter III. Assuming a density of about 10,000 lines per square centimeter in the armature iron, the core must have an area of $\frac{14 \times 10^6}{10{,}000} = 1{,}400$ square centimeters, or $\frac{1{,}400}{6 \cdot 45} = 220$ square inches, and since the magnetic lines have two paths through the core, each need be only 110 square inches in section. But the effective length of iron parallel to the shaft has been found to be 15·3 (see page 212). The depth of core must therefore be $\frac{110}{15 \cdot 3} = 7 \cdot 25$, and the slots are 1″, the total external diameter of the armature core will be $49\frac{1}{2} + 2\,(1 + 7\frac{1}{4}) = 66\frac{1}{4}''$. From this the weight of core can be found to be

$$\frac{\pi}{4}\left\{(66)^2 - (49\tfrac{1}{2})^2\right\} \times 15 \cdot 3 \times \cdot 28 = 6{,}500 \text{ lb.}$$

The watts lost per lb. at 25 periods and a magnetic density of 10,000 lines per square centimeter can be read off the curve given on page 61, and found to be 2·6. The total watts lost in the armature iron will be $6{,}500 \times 2 \cdot 6 = 17{,}000$. The total watts lost on the armature are thus $17{,}000 + 4{,}000 = 21{,}000$ and the cylindrical surface of the armature is $49\frac{1}{2}\,\pi \times 28$, 28″ being the length over the windings (see Fig. 104) $49\frac{1}{2}\,\pi \times 28 = 4{,}300$, and the watts lost per square inch are $\frac{21,000}{4,300} = 4{\cdot}8$. A loss of 5 up to even 6 watts per square inch calculated in this way is permissible for a rise of 70° F. above the surrounding air, and the armature should therefore be perfectly safe as regards heating.

The efficiency of the generator at full load and with ·8 P.F. can now be calculated—the losses are :—

Armature copper losses	4,000
Magnet copper losses	4,000
Armature Iron losses	17,000
Friction and windage (say)	3,000
	28,000

and the output of the machine is 300,000 watts, the total input must therefore be 328,000, and the efficiency becomes $\frac{300,000}{328,000} = {\cdot}91$ or 91%.

It must be noticed that the efficiency at full load, but with unity power factor will be somewhat better; the armature current will be only 86 amperes instead of 108, and the armature losses will thus be somewhat smaller also, as the armature back ampere-turns will be considerably reduced, fewer ampere-turns will be required on the magnets, and the losses on the magnet winding will also be reduced.

§ 8. **The Magnetic Circuit.**—The calculations for the magnetic circuit can be carried out in the same way as in Chapter III. In the C.C. generator the calculations were for the purpose of finding the required number of ampere-turns in order to put the necessary flux in the different parts of the magnetic current; in the present instance this has already been settled at 11,000 ampere-turns per pole in order to give the required regulation. The iron parts of the circuit have all been settled, but nothing as yet has been said as to the radial depth of the air gap; this dimension has now to be chosen such as to give 11,000 ampere-turns on each magnet pole.

The ampere-turns required on the armature core are so small that they may be neglected. From the table of magnetic densities and corresponding ampere-turns for different parts of the circuit given below, it appears that 1,060 ampere-turns are required on the magnet cores and yoke, and 390 ampere-turns on the teeth. This table is worked out in exactly the same way as the table on page 68. The length of path in the magnet cores and yoke is measured from a drawing to scale of the magnet system, such as that in Fig. 98. The armature back ampere-turns at full load and power factor ·8 are 2,200. These three items add up to $1{,}060 + 390 + 2{,}200 = 3{,}650$, which subtracted from 11,000 leaves 7,350 ampere-turns to be used up on the air gap. The density in the air gap at the pole face has been agreed upon as 8,000 lines per square centimeter, and it will be found that an air gap of $\frac{7}{16}''$ radial depth absorbs 6,800 ampere-turns at this magnetic density. Adding 10% for the bunching of the lines into the top of the teeth, this number is brought up to 7,480, which is sufficiently near the 7,550 ampere-turns found above.

The values for the different parts of the current can now be tabulated thus :—

	B	H	Length in Cms.	H × L	A-Ts.
Air gap .	8,000	8,000	$\frac{7}{16}''$ = 1·06 Cms.	8,500	7,480
Teeth .	{ 20,000 18,500	} 170	1″ = 2·54 Cms.	490	390
Magnets	15,000	35	38 Cms.	1,325	1,060
					8,930
		Armature Back Ampere-turns .			2,200
		Total . . .			11,130

This is sufficiently nearly equal to the 11,000 ampere-turns agreed upon, and it is seen from the table that 8,930 ampere-turns are required to put the flux through the different parts of the magnetic circuit, and that the remainder are required to neutralise the armature back ampere-turns.

§ 9. Magnetisation Curve and Voltage Regulation—Armature Reactance.—In order to draw a magnetisation curve of the machine, which is necessary in order to find the variation in E.M.F. when the load is thrown on or off, repeat the table of ampere-turns on page 218. Increasing the total flux, and therefore the terminal E.M.F. at no load in the ratio of say 8 to 9, it will now be found that about 11,270 ampere-turns are required to put the flux through the magnetic circuit. Again repeat the table, increasing the flux in the proportion of 8 to 9·4, and it will be found that 13,450 ampere-turns are required to put the flux through. This gives three points through which a curve can be drawn connecting the ampere-turns on the magnets with the terminal voltage of the machine at no load.

From the original calculation it is seen that 8,930 ampere-turns give at no load a terminal E.M.F. of 1,175 volts (see page 198); from the second and third tables suggested above 11,270 ampere-turns will give a voltage of $1,175 \times \frac{9}{8} = 1,320$, and 13,450 ampere-turns will give a voltage of $1,175 \times \frac{9{\cdot}4}{8} = 1,380$ volts. The curve in Fig. 105 is drawn through these three points, and represents the terminal voltage per phase at no load due to any given number of ampere-turns per pole on the magnets, and it has been found that at full load, the power factor of the circuit, being ·8, the ampere-turns required are 11,130. If the load be thrown off the 11,130 ampere-turns will give 1,320 volts (point *a* on the curve), the rise in volts will therefore be 1,320 – 1,150 = 170, and this expressed as a percentage of the full load E.M.F. is $\frac{170}{1,150} = \cdot 149$, or 14·9%, which is just within the specified rise allowed.

Sufficient data are now available to enable the reactance voltage of the armature to be calculated, from which the

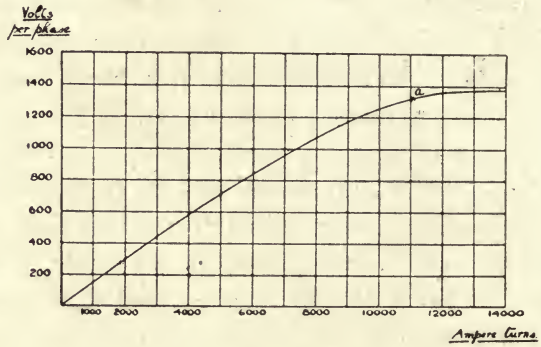


FIG. 105.

regulation to be expected with a non-inductive load can be arrived at.

Referring back to Fig. 96 and page 203, where it was stated that O B could not be calculated until the armature winding was settled; it is this quantity which it is now proposed to calculate. This will be done on the same lines as already given in Chapter III., page 80.

Calculating the reactance voltage for one phase, there are 10 turns per pole per phase and 8 poles; the total turns required per phase will be wound in 4 coils, each having 20 turns.

In Chapter III., it was assumed that one turn carrying one ampere was linked with 20 lines of magnetic force per inch length of core; this value should in the case of an alternator be increased to say 30 lines, because in the case of the C.C. generator the reactance of that coil only which was being commutated was under consideration,

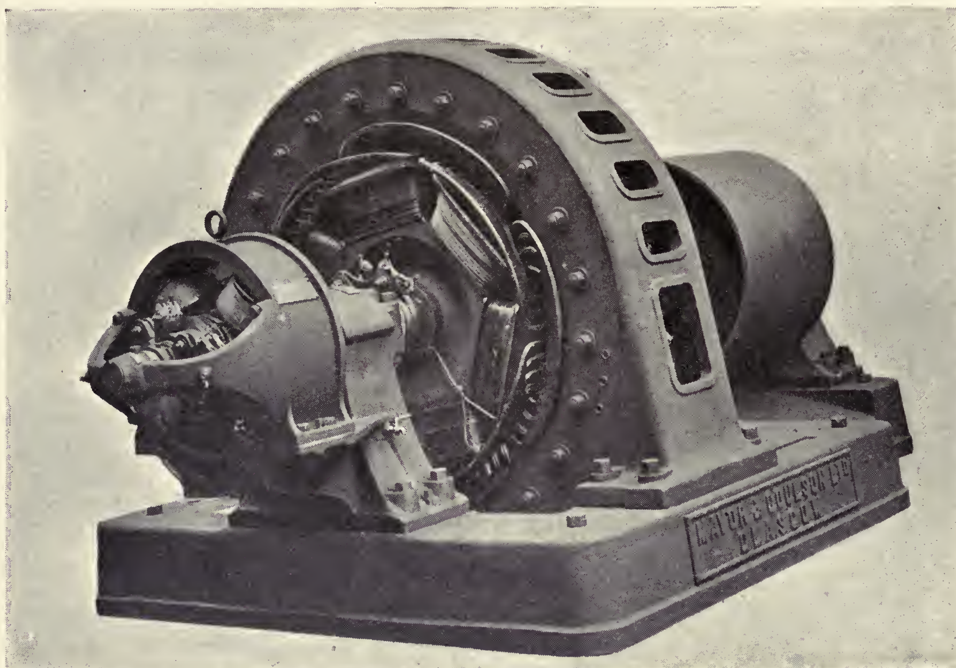


Fig. 106.—Three-Phase Generator.

and this coil is necessarily away from the pole shoes during the period of commutation. For the present case the reactance of all the armature coils is being dealt with, and some of these will be under pole shoes, and therefore there will be a much larger proportion of iron in the path of the magnetic lines surrounding them; the magnetic flux linked with the circuit will therefore be increased, and the higher value of 30 lines per inch length of core should be taken.

Each of the 20 turns in each coil, if carrying one ampere, will then be linked with $20 \times 30 \times 20 = 12{,}000$ lines, 20″ being the length of core, 30 the number of lines per inch length, and 20 the number of turns in one coil. The inductance of one coil is equal to the number of lines multiplied by 20, since the magnetic lines cut each of the 20 turns, and the result must be multiplied by 4, since there are 4 such coils in series in each phase winding; $\therefore$ $L = 12{,}000 \times 20 \times 4 \times 10^{-8} = \cdot 0096$, and the reactance is equal to $2\pi n L$, where n is the number of periods per second—in this case 25. Thus the reactance $= 2\pi \times 25 \times \cdot 0096 = 1\cdot 5$, and the reactance voltage is $1\cdot 5 \times 86 = 129$, the value of the full load current at unity power factor being 86 amperes.

The value of O B in Figs. 96 and 97 is therefore 129 volts, and from this it follows that in Fig. 96, which represents the conditions when the external load is non-inductive, the value of $\tan\phi$ is $\frac{129}{1{,}175} = \cdot 11$ and from a trigonometrical table it is seen that if $\tan\phi = \cdot 11$, $\sin\phi$ is also approximately $\cdot 11$. The armature back ampere-turns, with 86 amperes per phase being taken from the machine at unity power factor, will thus be $\cdot 11$ of the full armature ampere-turns per pole. These are equal to $\frac{10 \times 86}{\cdot 707} \times 2 = 2{,}450$ (pages 204, 205), and the back ampere-turns are therefore $2{,}450 \times \cdot 11 = 260$, or not much more than one-tenth of the back ampere-turns when the power factor is ·8.

Looking again at the curve, Fig. 105, there will be required at full load only 9,190 ampere-turns per pole on the magnet, 8,930 to put the magnetic flux through the circuit, and 260 to oppose the armature back ampere-turns, $8{,}930 + 260 = 9{,}190$, and if the load is thrown off, the speed and excitation remaining the same, the terminal E.M.F. will rise to 1,200 volts per phase; a rise of 50 volts $\frac{50}{1{,}150} = \cdot 043$, or 4·3%, which again is within the rise of 5% allowed by the specification.

The very great importance of voltage regulation in the design of an alternator must be kept in mind, and a careful comparison of the methods of this chapter with those of Chapter III. will show that whilst exactly the same general principles are involved, the relative importance of different points of the design varies in the two cases, and that it is in consequence convenient considerably to modify the point of view from which the design is attacked, and the order of the various calculations.

Fig. 106 shows the general appearance of an alternator of the type discussed in this chapter, together with a direct-coupled exciter.

CHAPTER IX

THREE-PHASE INDUCTION MOTOR

§ 1. General Construction of Induction Motors.—The calculations necessary for designing an induction motor whilst on the same general lines as those already made use of in the case of continuous-current machines and alternating-current generators, nevertheless differ in important particulars.

The question of heating is still one of the most important and the losses, copper and iron, must be kept down so that the temperature-rise is not greater than specified. As there is no commutator there is no question of sparking to be considered, and the designer is therefore relieved of the restrictions which limit his freedom of choice in continuous-current work from this cause. On the other hand, the power factor of the motor, the current taken at no load, and the question of the starting torque and of the breakdown torque, are new features which require attention. The following table shows the calculation required for the design of a three-phase induction motor :—

Heating	Starting	Breakdown
Copper losses in the stator . . .	Starting torque	Breakdown torque
Copper losses in the rotor . . .	Corresponding current	Corresponding current
Iron losses in the stator . . .		
Iron losses in the rotor		

The polyphase induction motor usually consists of a fixed part known as the stator, made up of annular iron stampings, which are held together in a cast-iron frame; slots are punched round the internal periphery, through which are threaded the copper wires which form the winding. Inside the stator is placed a rotor which also consists of iron discs threaded on to a spider, the construction being much the same as that of a continuous-current dynamo armature. These discs are also slotted on their external periphery, and the slots contain the copper windings.

The winding in the stator slots is connected to the supply mains; the rotor winding, on the other hand, is not connected to any source of supply, but the currents flowing in it are induced by the magnetic flux due to the currents in the stator winding. In fact, for many purposes, it is useful to consider the induction motor as a transformer, the stator winding acting as the primary and inducing an E.M.F. in the secondary winding, which is carried by the rotor.

As has been seen in Chapter VII., the effect of polyphase currents, passed into a suitable winding, such as the winding of the stator, is to produce a rotating magnetic field, and the lines of force of this magnetic field, as they sweep round the rotor, will induce in it currents, the direction of which will be such as to oppose the motion of the field. The reaction between the currents and the field will carry round the rotor with increasing speed in the same direction as the revolving field. If, however, the speed of the rotor becomes as great as that of the field, there will be no cutting of the conductors by the lines of force, there will be no E.M.F.s generated, and no currents flowing in the rotor conductors. This speed, which is called the synchronous speed, can therefore only be attained under the supposition that there is no load whatever on the rotor, and that no force is required to keep it running at synchronous speed.

This, of course, is unattainable in practice, as there will always be bearing and air friction to be overcome; these, however, will be so small that the rotor attains, when unloaded, to very nearly synchronous speed. If more load is put on to the motor, more current must flow in the rotor circuits, and this can only be got by the rotor pulling up slightly, so that the relative velocity of the field and rotor conductors becomes greater, the E.M.F. being thereby increased and also the rotor current.

The connections of the rotor bars to one another are evidently of no very great importance; in fact if a metallic cylinder be arranged so as to be capable of rotation within the stator, currents will be generated in this cylinder sufficient to make it run round and at the same time drive a considerable load. It is found, however, that such a simple device as this is not very efficient; of the currents generated in the metal sheet many are not in the best position to give a good torque, and the plan is therefore universally adopted of building up an iron core with slots, and threading in copper bars. A very usual method is to connect all these bars together at each end by means of a copper ring which is riveted, or in some way fastened, to the end of every bar. The currents find their way along the bars under one pole, along the end ring and back by bars under a pole of opposite polarity.

Such a construction is called a squirrel-cage rotor, it is the simplest possible system of connections and, owing to the absence of slip rings and moving contacts, is a most excellent device whenever possible.

Whenever an induction motor can be started on a light load, a squirrel-cage rotor answers the purpose admirably. It is, however, found that if it is required to start against a torque in any way approaching full load, a squirrel-cage rotor takes a very excessive starting current. It is necessary under such conditions to be able to introduce resistance in the rotor circuit which can be gradually cut out as the motor accelerates, and for this purpose it is necessary that the rotor should have one or more windings carried out to slip rings to which the resistance and the necessary switch are connected. A convenient way of carrying out this winding, is to wind the rotor with a three-phase star-connected winding; three rings are then all that are necessary to introduce resistance in each branch, and rotors are usually wound in this way whether they are intended for use in a three-phase, two-phase, or single-phase stator.

When a wound rotor is used, it is of course necessary that the copper should be insulated from the iron of the core as carefully and thoroughly as in the case of any winding which is to be connected to an external source of supply. In the case of the squirrel cage, however, a slight insulation in the slots suffices, and it is not unusual to connect both of the copper end rings to the body of the rotor.

It has so far been assumed that the stator carries the primary winding, that is to say, that the winding on the stator is connected to the source of supply whilst the rotor carries the secondary winding, which does not necessarily have any external connections, and has its current induced by the magnetic flux due to the primary. Theoretically there is no reason why this arrangement should not be reversed, and the primary winding put on the rotor, the secondary on the stator. In practice, however, the first arrangement is in all ordinary cases adopted. The current in the primary being drawn from an external source of supply, if this primary is put on the revolving member, there must of necessity be slip rings to lead it into the winding. If, however, the primary is put on the stationary member, the current can be led straight into its terminals, and if the conditions allow of the use of a squirrel-cage rotor, no slip rings at all are required. Moreover, the E.M.F. on the secondary can always be kept down to whatever limits may be chosen by a simple arrangement of the windings. The E.M.F. on the primary, on the other hand, is fixed by the source of supply. It is obviously better that the low-voltage current should be the one which has to pass through movable contacts, and this is another reason in favour of putting the primary winding on the stator.

§ 2. Calculations of Stator Dimensions for a 100 H.P. Three-Phase Motor.—It is proposed to design a three-phase motor to give 100 H.P. when running at 500 revolutions per minute on a circuit of 500 volts and 25 periods.

It is necessary in the first place to calculate the current per phase required to give this; 100 B.H.P. is equal to an output of 74,600 watts, and the required input will depend on the efficiency of the motor and its power factor, and as these can only be determined after the windings have been designed, calculations must in the first instance be based on an assumed efficiency and power factor. For a motor to give the above output an efficiency of 90% and a power factor of ·92 may be assumed.

The input will then be $\frac{74{,}600}{\cdot 9} = 83{,}000$ watts, and the volt-amperes will be $\frac{83{,}000}{\cdot 92} = 90{,}000$ volt-amperes.

The windings of the three phases being symmetrical, and in every respect alike, the input into each phase will be one-third of the total, that is, 30,000 volt-amperes must be put into each of the phases. At this stage it is necessary to determine whether the motor shall be star-connected or delta-connected. In the former case the voltage at the terminals being 500, the voltage per phase will be $\frac{500}{1 \cdot 73} = 290$, and the current per phase will therefore be $\frac{30{,}000}{290} = 103$. In the case of a delta connection the voltage per phase is equal to that of the terminal, 500 volts, and the current per phase will therefore be $\frac{30{,}000}{500} = 60$.

If a star connection is decided upon, each winding will have enough turns to give a back E.M.F. of 290 volts only; if a delta connection, the conductors may be of smaller section as they only have 60 amperes to carry, instead of 103, but more of them will be required, since they have to give a back E.M.F. of 500 instead of 290 volts.

Whether star or delta winding shall be adopted is a question of convenience in winding. In small motors for high-voltage circuits, where many small conductors are required, there is an advantage in a star connection, using therefore conductors of the greatest possible section, and having as little room as possible lost in insulation. In larger motors the question must be considered in each case so as to give the most convenient winding. It is proposed that this motor shall be star-connected.

The number of poles is fixed by the speed and the periodicity $\frac{25 \times 60}{500} = 3$, that is, the number of pairs of poles must be 3, the number of poles 6. The output coefficient (watts per revolution) of the machine is equal to $\frac{90{,}000}{500} = 180$. From the curve, Fig. 107, the corresponding value of $d^2 l$ is 8,100; this will be satisfied by taking 30″ as the diameter, and 9″ as the length of the rotor core. The curve connecting D and $d^2 l$ differs, it will be observed, from that used for C.C. machines; the value of $d^2 l$ for a given output is considerably greater in the induction motor.

Thus far it has been found that a motor to give 100 B.H.P. at 500 revolutions on a 25 ∿, 500-volt circuit must have 6 poles and, if star-connected, a stator winding adapted to give a back E.M.F. of 290 volts, and to carry a current of 103 amperes per phase. Also the stator should have an inside diameter of about 30″, and a core length of 9″.

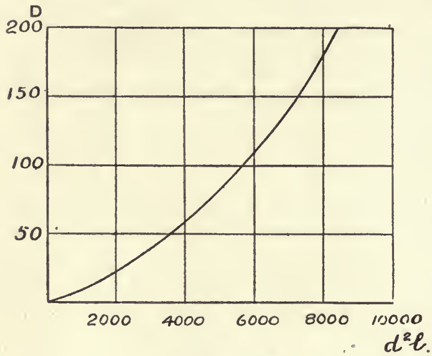


FIG. 107.

§ 3. **The Circle Diagram.**—The losses in such a motor will be found on much the same lines as in Chapter III., and from the losses the probable heating and the efficiency can be deduced, but the other properties of the motor—the breakdown current and torque, and the starting current and torque—are best studied by means of what is known as the circle diagram. As this circle diagram depends to some extent on the number of bars in the winding, it will be advisable to consider it before proceeding to determine the number of slots and bars on the stator.

To predict fully the performance of a motor under different conditions of load, it is required to know (1) how many amperes will flow in the primary circuit, (2) what will be the phase relation between the current and the E.M.F., and (3) what will be the speed corresponding to any given torque.

The relations between these different quantities are very complicated, and a full mathematical investigation involves long and complex expressions. Approximately correct results are, however, easily obtained graphically by neglecting some quantities of secondary importance.

If the resistance of the primary winding and the current required to supply the iron losses in the stator and rotor cores are left out of account, a diagram such as Fig. 108 can be drawn for any motor.

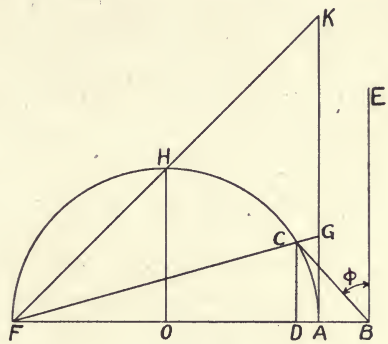


FIG. 108.

This diagram is known as the Heyland diagram, or, frequently, as the circle diagram of the motor. In order to draw it for any particular motor two quantities only are required to be known :—(1) A B, which represents to a convenient scale the number of amperes flowing through the primary circuit at no load (this current being referred to as the no-load or the magnetising current); (2) a quantity depending on the self-induction of the primary and of the secondary circuits, and usually denoted by U. The methods of calculating these quantities for any particular motor will be explained later; when they have been ascertained the diagram is constructed as follows:

Draw a straight line B F, and from it cut off B A representing the no-load current; make A F = U' × B A, and on A F describe the semicircle F C A, draw B E at right angles to F B, B E represents in direction the E.M.F. applied at the terminals of the primary. On the circle take any point C, then B C will represent the current flowing in the primary and C D the corresponding torque. The angle ϕ included between C B and B E will be the angle by which the current lags behind the E.M.F., so that cos ϕ will be the power factor of the motor for that particular load at which the torque = C D. At A erect the perpendicular A G, join F C and produce it to G, then A G is proportional to the slip at this particular load. The slip is the amount by which the speed falls short of synchronous speed, and is expressed as a percentage of synchronous speed.

For example a four-pole motor running on a circuit of 25 ∿ will have synchronous speed of $\frac{25 \times 60}{2} = 750$ revolutions per minute; if at full-load this motor runs at 730 revolutions, the slip will be $\frac{750 - 730}{750} = \cdot 026$ or $2 \cdot 6\%$.

Thus if in Fig. 108 the point C correspond to full load, C D will represent the full-load torque, C B full-load current, and cos ϕ the power factor of the motor, at full load. As the load decreases, the point C moves down the circle towards A, the slip represented by A G becomes less, that is, the speed increases and approaches more nearly to synchronous speed; the power factor also decreases, because as the angle ϕ increases, its cosine decreases in value. When the point C coincides with A, the load is nothing, since the torque represented by C D has vanished, and the slip AG is nothing; the motor is therefore running at synchronous speed, the current is represented by A B, and it is to be noticed that it is at right angles to the E.M.F.; the current is entirely out of phase with the E.M.F. since cos $90^\circ = 0$. The primary current at no-load is entirely used in putting the magnetic flux through the magnetic circuit, and it is therefore indifferently known as the no-load or as the magnetising current.

If, on the other hand, the load is increased above full-load torque, the point C will move up the circle towards the point H, the current and slip will increase—that is, the speed will fall farther away from synchronous speed—and the motor running at a slightly lower speed will go on carrying an increasing overload, until the point H is reached; H O represents the torque. Since H O is the longest perpendicular which can be erected in the semicircle, it represents the greatest torque which the motor is capable of giving out. If the load is increased beyond this point, the motor is incapable of giving the torque required of it, and comes to rest. For this reason OH is called the breakdown torque of the motor, and the ratio $\frac{OH}{CD}$ is called the breakdown factor. Suppose $\frac{OH}{CD} = 2$, this will mean that the motor is capable of giving a torque equal to twice full-load torque, but that if any greater load than corresponds to this is put upon it, it will immediately pull up and come to a standstill.

§ **4. Proof of the Circle Diagram.**—The proof of the circle diagram can be geometrically deduced from the vector diagram of the motor, and this in its turn is most easily arrived at, by starting from the different magnetic fluxes which interlink with the two windings.

In Fig. 109, P represents a section of a conductor in a primary circuit, S a conductor parallel to P, and acting as a secondary, four lines are drawn round these, showing the general direction of the paths of magnetic lines of force. Of the lines of force due to the current in P, a part will surround P only, but will not also interlink with S; these are denoted by the line PL: in the induction motor they are called leakage lines, because they leak along the air gap and other stray paths, but do not cut the secondary conductors, and therefore do no useful work. The greater part of the magnetic

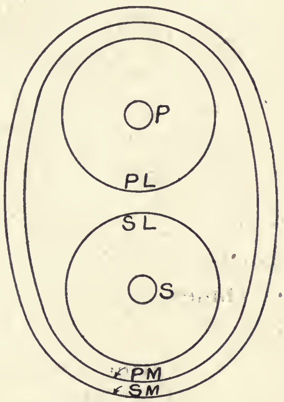


FIG. 109.

field will, however, include both circuits; this is denoted by P M. It must be clearly understood that P M represents the magnetic field due to the current in P only; it is the field which would be present if there were no current in S. Similarly, the field due to the current in the secondary can be divided into leakage flux, S L, which links with the secondary only, and main flux S M, which encloses both secondary and primary.

Again, S M is the field which would be caused by a current in S on the assumption that no current was flowing in the primary. When current is flowing in both circuits, the field actually present surrounding both conductors is the resultant of P M and S M. The leakage flux for any given motor may be calculated with more or less accuracy by assuming leakage paths and comparing the magnetic reluctance of these combined paths with the reluctance of the main magnetic circuit; a simple formula for doing this will be discussed in a future paragraph.

Whatever method is used for estimating the leakage flux, it is convenient to express it as a fraction of the useful flux. This method was used in the continuous-current machines by fixing on a leakage coefficient; in the case of the induction motor there will be two such coefficients, one applying to the primary, and one to the secondary. Every effort is made in induction motor design to keep down the leakage to as low a value as possible, and whilst leakage coefficients of the order of 1·2 to 1·3 obtain in C.C. machines, in the induction motor they are more in the nature of 1·02.

In drawing the clock diagram for an induction motor, three magnetic fields have to be considered, (1) the main flux which cuts both windings; this is the resultant of P M and S M (Fig. 109); (2) the total flux which cuts the primary; this is the resultant of the main flux and the primary leakage flux P L; (3) the total flux cutting the secondary, which is the resultant of the main flux and of the secondary leakage flux S L. These three magnetic fields will vary in magnitude and in their phase relation with different loads, but in all circumstances the following relations must hold. The total flux cutting the primary must have a constant value so long as the applied E.M.F. is constant. This follows from the fact that the back E.M.F. must (neglecting stator winding resistance) be equal and opposite to the applied E.M.F.; if the latter is constant, the field cutting the primary which is the sole source of back E.M.F. must also be constant.

The total field cutting the secondary will vary in magnitude with the load, but whatever its magnitude it must always be represented by a vector, at right angles to that representing the secondary current, because the cutting of the secondary winding by this field is the sole source of E.M.F. in the secondary winding.

In drawing the vector diagram of an induction motor, the convention is adopted that the vectors representing currents represent also the fields due to these currents, and that a magnetic field gives rise to an E.M.F. represented by a vector at right angles to the vector representing the field. As a matter of fact, the magnetic field due to any current is on the motor displaced by 90° from the current producing it, and by its motion generates an E.M.F. in phase with itself. But the above convention makes the diagram simpler by using only

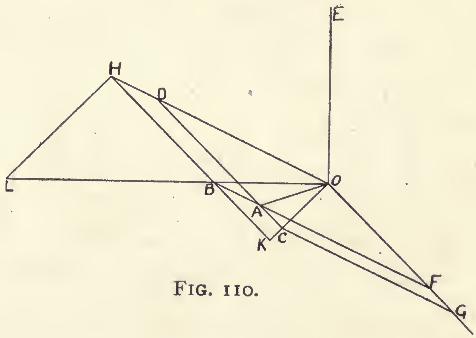


FIG. 110.

one vector for current and field instead of two, and it is easy to remember that all the magnetic fields are really displaced on the motor by 90° from the direction of the vectors representing them.

In Fig. 110, the vector diagram of an induction motor is drawn, the values of u_1 and u_2, the leakage coefficients, being exaggerated to avoid confusion in the diagram.

Draw O A to a convenient scale to represent the magnetic flux common to both circuits. The current flowing in the secondary will depend on the load, and can be represented by O G, the length of O G depending on the load only. From O G cut off O F, so that O G $= u_2$ O F. Then O F represents the flux (S M of Fig. 109) due to the secondary, which cuts both circuits, and O A is the resultant of the fields S M and P M. Therefore completing the parallelogram O D A F, O D represents the flux P M of Fig. 109. Produce O D to H, so that O H $= u_1$O D, then O H represents the resultant of the fluxes P M and P L, and also the primary current by which these fields are produced.

Completing the parallelograms O H B F and O D C G, the resultants O B and O C can be drawn which represent the total fields cutting the primary and the secondary respectively. That this is so is plain on considering that in Fig. 109 the resultant of P M, P L, and S M is the total field cutting the primary, and that the resultant of S M, S L and P M is the total field cutting the secondary, whilst in Fig. 110, O D represents the field P M, D H the field P L, O F the field S M, and F G the field S L.

Again, since the E.M.F. generated by any flux is shown on the diagram by a vector at right angles to the field vector, it is evident that the applied E.M.F. which is equal and opposite to the back E.M.F. in the stator, must be represented by a line at right angles to O B : O E represents the applied E.M.F. It is necessary that O C be at right angles to O G, since O C represents the total field cutting the secondary, and O G represents the current in the secondary which is necessarily in phase with the total E.M.F. generated in the circuit. If in the construction O C does not come at right angles to O G, it is because the proper phase angle has not been originally chosen for fixing the position of O G. The length of O G can be chosen arbitrarily, as it depends only on the load, but for any length chosen there is a fixed position of O G, relative to O A, which depends on the values of u_1 and u_2, and which will bring O C at right angles to O G. If this has not been correctly chosen in the first instance, the diagram must be modified until O C and O G have their proper relation at right angles to one another.

Produce O C and H B to meet at K. Produce O B to L, making $LB = U \times OB$ where $U = \frac{1}{u_1 u_2 - 1}$. Describe a semicircle on L B. Whatever value be chosen for O G, that is, whatever the load on the motor, the point H will always be on the semicircle.

$CD = u_2 \times AD$ because $CD = OG$ and $AD = OF$, and $BH = AD$, $\therefore CD = u_2 \times BH$, again because the triangles O H K and O D C are similar;

$$\therefore \frac{KH}{CD} = \frac{OH}{OD}, \text{ but } OH = u_1 \times OD, \therefore \frac{OH}{OD} = u_1,$$

$$\therefore KH = u_1 \times CD, \text{ and } CD = u_2 \times BH,$$

$$\therefore KH = u_1 \times u_2 \times BH, \text{ and } KB = KH - BH,$$

$$\therefore KB = u_1 \times u_2 \times BH - BH = (u_1 \times u_2 - 1)\,BH,$$

$$\therefore BH = \frac{KB}{u_1 u_2 - 1} \text{ and } \frac{1}{u_1 u_2 - 1} = U,$$

$$\therefore BH = U \times KB, \text{ and by construction } LB = U \times OB,$$

$\therefore$ the triangles B K O and B H L are similar, and B K O is a right angle, because it is equal to K O G. It has been seen that K O and G O are always at right angles, $\therefore$ B H L is a right angle, and, therefore, H moves on the semicircle drawn on BL as diameter.

Moreover, the field OB is constant whatever the load, because it supplies the back E.M.F. which is opposite and equal to the applied E.M.F., OE, which latter is itself constant. Again OB is the resultant of the primary current OH and of OF; if there be no load on the motor OG, and OF become zero, it is evident that the primary current OH must then coincide with OB; therefore OB represents the no-load current of the motor.

If $\sigma = \frac{1}{U}$, then σ is called the dispersion coefficient of the motor, and since in the circle diagram $LB = U \times OB$, it is evident that $LB = \frac{OB}{\sigma}$.

§ 5. **The Magnetising Current.**—Returning now to the 100 H.P. motor, specified at the beginning of this chapter, the relation between the dimensions of the machine, and the constants required to draw the circle diagram may be investigated. These constants, as has been seen, are two in number: the magnetising current to which OB (Fig. 108) is proportional, and the dispersion coefficient σ by which OB must be divided in order to give BL (Fig. 110) the diameter of the circle.

Both these constants are to a large extent dependent on the radial depth of the air-gap. Since the greater part of the reluctance of the magnetic circuit lies in the air-gap, the magnetising current necessary to produce the main flux will vary almost directly as the depth of air-gap, and since a large proportion of the leakage lines have to pass along the gap, the smaller this is kept the larger will be the reluctance of the leakage paths, and therefore the smaller the leakage.

From both points of view, therefore, the air-gap should be made as small as possible, and mechanical considerations alone determine the minimum depth which can be used. By careful attention to the construction it is quite possible to work such a machine as this with an air-gap not exceeding $\frac{1}{16}''$ a side. Say $1\frac{1}{2}$ millimetre.

Another dimension of great importance is the pole pitch measured round the inner circumference of the stator. Since the diameter is 30″, and the number of poles 6 this is equal to $\frac{30\,\pi}{6} = 15{\cdot}5$. The pole area available for carrying the flux will then be 15·5, multiplied by the effective length of iron in the stator core. The gross length is 9″ (see page 229), and the length of iron measured parallel to the shaft will be 9″ less, say, 2″ (the allowance for 4 ventilating ducts each $\frac{1}{2}''$ wide), leaving 7″ for discs, and insulation between them, or $7 \times {\cdot}9 = 6{\cdot}3''$, the effective length of iron. The air-gap area is therefore $15{\cdot}5 \times 6{\cdot}3 = 97{\cdot}5$ square inches = 620 square centimeters. As, however, some of the iron will have to be cut away in order to provide the slots in rotor and stator, part only of this will be effective. If the slots both in the stator and rotor are made fully closed the whole of the area as calculated above is available, but totally-closed slots increase the leakage by giving an iron path in addition to the path along the air-gap; they therefore increase the value of σ, and this gives an undesirably small circle. It is therefore advisable that the slots should be at any rate partly open.

As a compromise the slot openings may be made of such width that the effective area of the air-gap is 80% of the total area. If this be done in the present case, $620 \times {\cdot}8 = 490$ square centimeters will be obtained as the area available at each pole for carrying the total flux.

The number of conductors required in the stator winding is settled by the consideration that whilst the back E.M.F. to be generated in each phase is 290, the total flux producing this back E.M.F. must be kept down to such limits that the magnetising current required to put it through the various parts of the magnetic circuit is not too great a proportion of the full-load current. It is usual to aim at a value of the magnetising current between 25% and 35% of the full-load current. In extreme cases values as high as 40% have been used.

A formula will be given later for calculating directly the number of turns per pole per phase required in the stator winding in order to give any desired value of the magnetising current. But in the first place a number of turns, 15 per pole per phase, will be assumed, and the resulting magnetising current calculated, as this process will most readily show the steps by which the formula is arrived at.

If there are fifteen turns per pole per phase, and 6 poles on the machine, the number of turns per phase will be $6 \times 15 = 90$, and the back E.M.F. to be generated in these 90 turns is 290, the value of N. The total flux per pole is therefore obtained from the formula

$$N = \frac{290 \times 10^8}{4{\cdot}2 \times 90 \times 25} = 3{\cdot}08 \times 10^6$$ (see Chapter VII., page 193) if a value of ·95 be assumed for q, the breadth coefficient.

But the effective area of the air-gap is 490 square centimeters, and the value of B the induction will therefore be $\frac{3{,}080{,}000}{490} = 6{,}250$. The ampere-turns required to put this flux through an air gap of ·15 centimeters

$$= 6{,}250 \times {\cdot}15 \times \frac{10}{4\pi} = 750.$$

The ampere-turns required for the iron parts of the circuit are usually so small that it is considered sufficiently accurate to add a certain percentage, say 15 %, to the ampere-turns required for the air-gap, and to call this the total ampere-turns required for the whole circuit. If the inductions in either the teeth or the stator and rotor cores are unusually high, this percentage may be increased to 20%.

On the assumption that 15 % will cover the ampere-turns required for the iron parts of the circuit the total ampere-turns required will be $750 \times 1{\cdot}15 = 860$. These ampere-turns will be provided by the three phases; divide by 2, $\frac{860}{2} = 430$ ampere-turns to be provided by the winding of each phase (see page 205). But each phase has 15 turns per pole and the current flowing through in order to give 430 ampere-turns, is therefore $\frac{430}{15} = 29$ amperes and the full-load current of the machine is 103 amperes; the magnetising current is therefore $\frac{29}{103} = {\cdot}28$ or 28% of the full-load current.

If t represents the number of turns per pole per phase, a general expression for finding t can be obtained by repeating the above process in symbols instead of figures.

E = E.M.F. per phase,
P = number of poles,
$\sim$ = periodicity,
C_1 = magnetising current per phase,
A = effective area of one pole,
Δ = radial depth of air-gap,

$$\text{then } N = \frac{E \times 10^8}{4{\cdot}2 \times t P \times \sim}$$

$$\text{and } B = \frac{N}{A} = \frac{E \times 10^8}{A \times 4{\cdot}2\, t P \times \sim};$$

again ampere-turns required

$$= B \times \Delta \times \frac{10}{4\pi} \times 1{\cdot}15$$

$$= \frac{E \times 10^8 \times \Delta \times 10 \times 1\cdot 15}{A \times 4\cdot 2 \times t \times P \times \sim \times 4\pi}$$

$$= \cdot 22 \times 10^8 \frac{E \times \Delta}{A \times t \times P \times \sim},$$

and the ampere-turns per phase are half of this or $\cdot 11 \times 10^8 \frac{E \times \Delta}{A t P \sim}$, but again if C_1 be the magnetising current $C_1 \times t$ = ampere turns actually on each phase,

$$\therefore C_1 \times t = \cdot 11 \times 10^8 \frac{E \times \Delta}{A t P \sim},$$

$$\therefore t^2 = \cdot 11 \times 10^8 \frac{E \times \Delta}{A C_1 P \sim}.$$

This is the general formula enabling the value of t to give any required magnetising current to be calculated directly. Applying this to the present case, say that it is determined that the magnetising current should be 30 amperes, about 30% of the full-load current. Substituting

$$t^2 = \cdot 11 \times 10^8 \frac{290 \times \cdot 15}{490 \times 30 \times 6 \times 25} = 220,$$

and the square root of 220 being very nearly 15 (15 × 15 = 225), this number would therefore be chosen.

The higher the magnetising current allowed, the fewer turns need be put on the armature, and therefore the smaller the machine may be made; on the other hand, a high magnetising current means a bad power factor, especially at low loads, and therefore a largely increased current for any given horse power.

§ 6. Copper and Iron Losses and Heating.—The number of turns which it is necessary to put on the stator having now been ascertained, it will be well, before proceeding to deal with the other constant required to draw the circle diagram, to ascertain first whether the required number of bars can be put on a stator of the given dimensions without undue heating.

The number of slots per pole per phase should be a whole number 3, 4, or 5, or more, according to the number of poles. In this case if 5 slots per pole per phase be chosen there will be a total of $5 \times 3 \times 6 = 90$ slots, and 15 turns per pole per phase are required on the stator. For the three phases and 6 poles this gives 270 turns or 540 bars, and as there are 90 slots, 6 bars will have to be put in each slot.

The current to be carried by each bar is 103 amperes, and for a first approximation to the size of the bar, a current density of about 2,200 amperes to the square inch may be chosen. A stranded conductor of 7/13s S.W.G. copper wire has a section of ·046 square inch, which gives a current density of $\frac{103}{\cdot 046} = 2{,}250$ amperes per square inch. It is necessary to calculate how many watts will be lost in a winding consisting of 180 bars per phase, each bar ·046 of a square inch, and carrying 103 amperes.

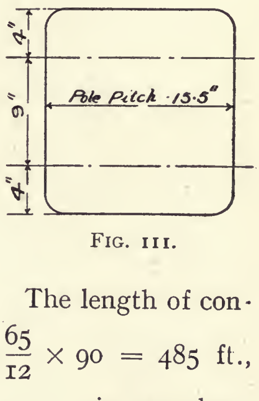


FIG. 111.

The length of one turn is best ascertained by setting out each coil to scale as shown in Fig. 111, from which it appears that the mean length of one turn is

$$2(9 + 15 \cdot 5 + 8) = 65'',$$

and in one phase there are 90 turns. The length of conductor in one phase will therefore be $\frac{65}{12} \times 90 = 485$ ft., and the resistance of 485 ft. of 7/13s S.W.G. is ·095 ohm; the watts lost in one phase are thus $(103)^2 \times \cdot 095 = 1{,}030$, and in the three phases 3,090.

For a preliminary calculation the losses in the rotor winding may be taken as equal to those in the stator. The iron losses will be chiefly found in the stator. The speed of the rotor being nearly equal to that of the rotating field, the periodicity in the rotor is very small. If the slip amount to 3% or 4%, the periodicity in the rotor will be only 3% or 4% of the impressed stator periodicity. In the present case it will be only about ·5 to 1 per second. Iron losses due to such a low periodicity will evidently be small, and attention may be chiefly directed to the stator losses. The losses may be divided into hysteresis and eddy current losses, and it might at first appear that the curve Fig. 30, Chapter III., could be used in this case as it was in the case of the continuous-current generator and motor. It should be noted, however, that the latter curve is entirely empirical, and based on experimental results in continuous-current machines. The distortion of the field due to armature reactions in machines of the types considered in Chapters III., IV., and VIII., cause the magnetic inductions in some parts of the iron to be very much higher than the average calculated values, and allowance is made for this in the curve of watts lost. Such distortions of the field are absent in the induction motor, and it is therefore quite safe to work from a curve showing much smaller losses. Such a curve is given in Fig. 112. The ordinates represent B × cycles per second divided by 1,000, the abscissæ watts lost per lb.

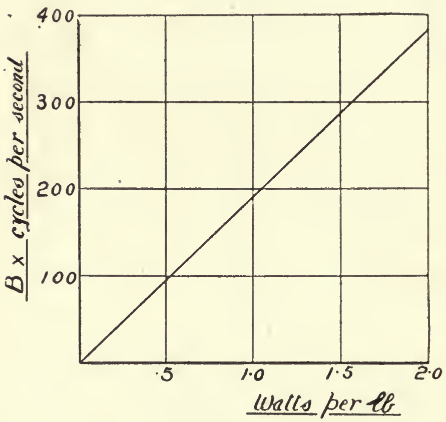


FIG. 112.

The induction B in the stator iron should be kept about 10,000 lines per square centimeter, and the total flux per pole (N) is $3{\cdot}08 \times 10^6$. The area of the stator should, therefore, be not less than 308 square centimeters, say 48 square inches.

Again notice that the flux from one pole divides into two paths, one to the right, and one to the left; the area of the core at any one section need therefore only be 24 square inches, and the effective length has already been found to be 6·3″. The depth above the slots must therefore be about $3\frac{3}{4}''$. Add to this 1″, the slot depth, and the radial depth of the stator on each side may be $4\frac{3}{4}''$, that is, the outside diameter of the stator discs must be $30 + 9\frac{1}{2} = 39\frac{1}{2}$. The total weight of iron will $\frac{\pi}{4}\left\{(39\frac{1}{2})^2 - (30)^2\right\} 6{\cdot}3 \times {\cdot}28$ $= 930$ lb., and 1·3 watt is lost per lb. (this is taken off the curve Fig. 112). The total watts are therefore 1,200. Add to this the stator $c^2\,\omega$ losses, and the rotor $c^2\,\omega$ losses, and the total losses on the whole motor are obtained, $1{,}200 + 3{,}100 + 3{,}100 = 7{,}400$. The surface of the stator calculated over the end windings is $30\,\pi \times 17 = 1{,}600$. The watts per square inch are therefore $\frac{7{,}400}{1{,}600} = 4{\cdot}55$.

A well-ventilated motor can easily get rid of 5 watts per square inch of surface calculated in this way and therefore the surface is ample; the design so far may be considered satisfactory from the point of view of heating.

§ **7. The Dispersion Coefficient.**—The value of the dispersion coefficient must now be found in order that the circle diagram of the motor may be drawn. It has been shown in § 4 (page 238) that σ is equal to $u_1\,u_2 - 1$, where u_1 and u_2 are the leakage coefficients for the primary and secondary respectively. As it is extremely difficult to estimate u_1 and u_2 accurately, it is proposed to deal with them not separately but together.

It was first proposed by Behrend that the value of σ should be considered equal to $c\,\frac{\Delta}{\tau}$, where c is a constant,

Δ the radial depth of the air-gap, and τ the pole pitch. The use of a formula in this form is justified by inspection of Fig. 113, which is a modification of Fig. 109 and shows the four leakage paths of the latter figure as they occur

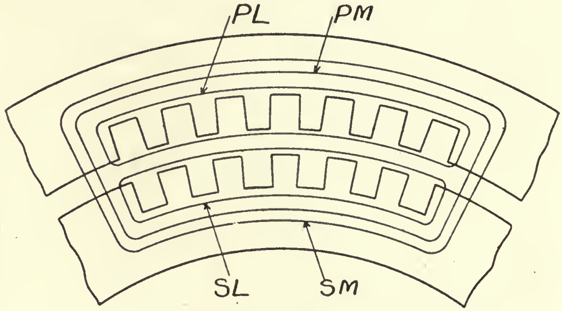


FIG. 113.

in an actual motor. The figure also shows the slots on a portion of the stator, and on that part of the rotor which is opposite to it.

Six slots are shown in the stator, and are supposed to occupy the whole of one pole pitch, that is, there would be in the stator two slots per pole per phase. Four lines are shown indicating the direction of the magnetic fluxes: P M the main flux, P L the leakage flux due to the current in the primary, S M and S L the main and the leakage flux due to the current in the secondary.

It is evident that since the leakage flux both of primary and secondary has to pass circumferentially along the air-gap, the reluctance of the leakage path will vary inversely at the section of the gap, that is, inversely as the radial depth of air-gap; and since the length of path is from one pole to the next, the reluctance will vary directly as the pole pitch; therefore the amount of the leakage flux calculated as a proportion of the useful flux will vary directly as τ and inversely as Δ. Thus by choosing a suitable value for the constant c in Behrend's formula, as given above, an approximately true value of σ will be arrived at.

There are, however, other leakage paths to be taken into account; the end windings for instance will have magnetic lines surrounding them which do not cut the other winding. The effect due to this cause is taken into account in a modification of Behrend's method proposed by Hobart. In this method the coefficient c in Behrend's formula is

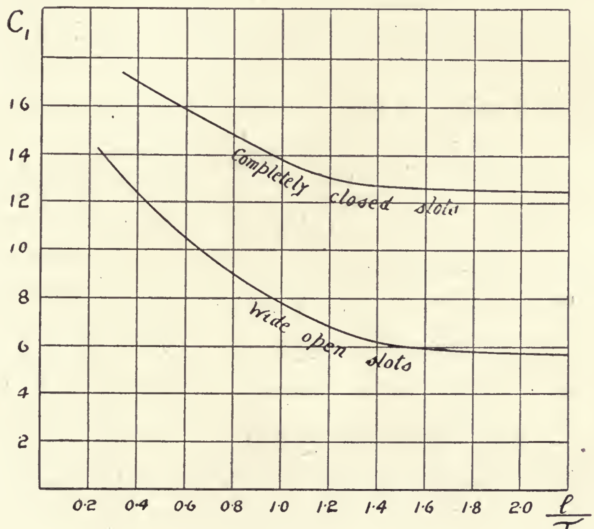


FIG. 114.

broken up into three coefficients, the values of which are found separately; thus Hobart's modification gives

$$\sigma = c_1 \ c_2 \ c_3 \ \frac{\Delta}{\tau}.$$

The coefficient c_1 is found from the curve given in Fig. 114, in which the value of c is made to depend upon the ratio $\frac{l}{\tau}$, l being the length of stator core and τ the pole pitch. As the length becomes greater in comparison to the pole pitch so does the effect of the leakage round the end windings become of less importance; the decreasing value of c_1 and therefore of σ given by the curve for high values of $\frac{l}{\tau}$ makes the necessary allowance for this end leakage. Two curves are given, one for fully open and one for entirely closed slots; the latter gives the higher value of c^1, because the leakage is increased when an iron path is provided along the air-gap, for the leakage lines. This can be seen in Fig. 113. If the slots are closed some of the flux PL and SL can flow along the iron bridging the slots, and the reluctance of the leakage paths will be decreased. If the slots are only partly closed, a point may be chosen intermediate between the two curves.

The coefficient c_2 is chosen of such a value as to make allowance for a third leakage which is known as zigzag leakage, this is shown in Fig. 115. When a stator tooth lies opposite a rotor slot, the lines of magnetic force will pass across the air-gap into the top of the tooth, and back to the other side of the slot, without cutting the rotor winding. This evidently increases the leakage coefficient of the rotor. In the same way when a rotor tooth magnetically short circuits a stator slot, the stator leakage coefficient will be increased. The zigzag leakage will be smaller the greater the radial depth of air-gap, and will also be decreased by having a large number of slots

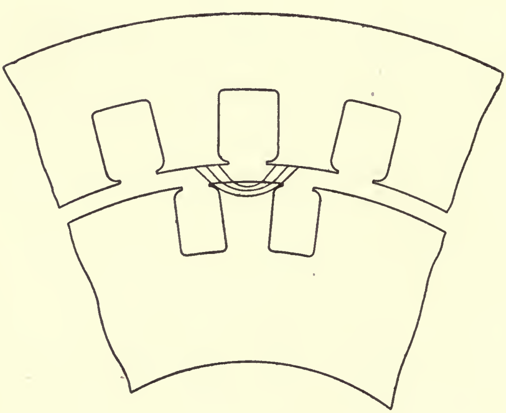


FIG. 115.

per pole. The curve, Fig. 116, connects values of c_2 with Δ M where Δ is the radial depth of air-gap measured in centimeters, and M is the average number of slots per pole counted on both stator and rotor.

Leakage in a motor having a squirrel cage rotor is considerably less than on the same motor if fitted with a wound rotor, and the third coefficient c_3 is introduced to make allowance for this; c_3 is taken as equal to unity for a wound rotor and equal to ·75 for a squirrel cage.

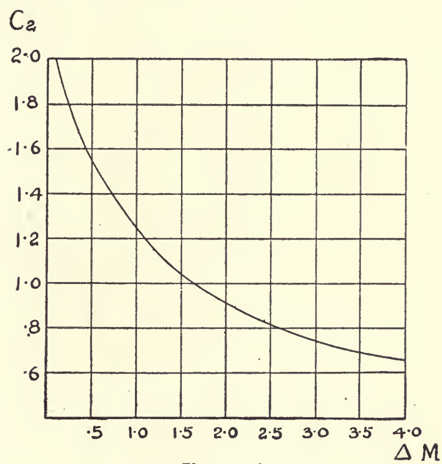


FIG. 116.

Applying the above to the 100 H.P. motor, the ratio $\frac{l}{\tau}$ is $\frac{6}{15 \cdot 5}$ = ·57. The curve, Fig. 114, gives the corresponding value for c_1 as 10·5 for open and 16 for closed slots. Since it has been decided that the slots of the stator should be partly open, an intermediate value may be chosen, say $c_1 = 13$. The number of slots per pole on the stator is 15, the number on the rotor has not yet been decided upon, it is usually chosen so as to be somewhat less than on the stator; say that the slots per pole on the rotor are to be about 13, then the mean number of slots per pole will be 14, and the depth of air-gap is ·15 centimeter, $\therefore \Delta \text{ M} = \cdot 15 \times 14 = 2 \cdot 1$. From the curve, Fig. 116, the corresponding value of c_2 is found to be about ·9. It was proposed that the rotor winding should be a squirrel cage, therefore $c_3 = \cdot 75$.

Again the depth of air-gap is ·15 centimeter, and τ

$= 15 \cdot 5'' = 39$ centimeters, therefore the value of the ratio $\frac{\Delta}{\tau} = \frac{\cdot 15}{39} = \cdot 0039$. The value of σ is thus obtained

$$\sigma = C_1\,C_2\,C_3\,\frac{\Delta}{\tau} = 13 \times \cdot 9 \times \cdot 75 \times \cdot 0039 = \cdot 034$$

and $U = \frac{1}{\sigma} = \frac{1}{\cdot 034} = 29 \cdot 5$.

§ 8. **The Breakdown and Starting Torque.**—All the data are now available for drawing the circle diagram of this particular motor.

The no-load or magnetising current is 29 amperes (see page 241), O A (Fig. 117), must therefore be drawn to scale to represent 29 amperes; the value of U is 29·5, therefore $29 \times 29 \cdot 5 = 860$ is the diameter of the circle. Draw A F, to the same scale as chosen for O A, to represent 860 amperes, and on A F describe the semicircle. The full-load current per phase is 103 amperes; choose the point C so that O C = 103 amperes. The point C will correspond to full load, and C D will represent full-load torque.

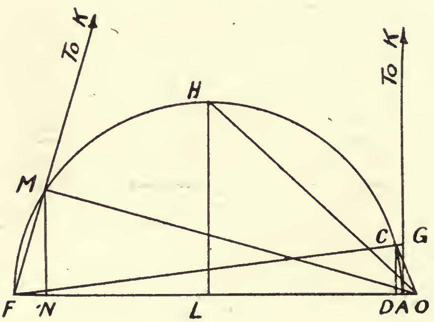


FIG. 117.

The motor at full load is giving 100 H.P., which is equal to $33{,}000 \times 100$ foot-pounds per minute, and the peripheral speed of any point on the rotor at 1 ft. distance from the centre is $500 \times 2\pi = 3{,}160$ ft. per minute. The torque is therefore $\frac{3{,}300{,}000}{3{,}160} = 1{,}030$ foot-pounds. If the diagram has been carefully drawn to scale, C D can be measured, and since it represents 1,030 foot-pounds torque, the scale to which torque is represented is thus found, and the torque at any other point can be ascertained.

If, for instance, the diagram has been drawn to the scale of $\frac{1}{16}'' = 10$ amperes of current, then all measurements being in $\frac{1}{16}$ths of an inch, OA $= 2{\cdot}9$, OC $= 10{\cdot}3$, and CD will be found by measurement to be about $9{\cdot}5$; the maximum torque of the motor is represented by HL, which is the radius of the circle, and therefore equal to half CF $\therefore$ HL $= \frac{86}{2} = 43$. Hence the breakdown torque of the motor is $\frac{HL}{CD} = \frac{43}{9{\cdot}5} = 4{\cdot}5$ times full-load torque and the breakdown current. The current corresponding to breakdown is represented by OH; if this be measured on the diagram it will be found to be $= 63$-16ths of an inch, and the breakdown current is therefore 630 amperes.

The current which the motor would take if switched on to the mains when at rest and the corresponding torque are known as the standstill current and standstill torque. To find by means of the circle diagram, what the values of current and torque at standstill will be it is necessary to know what is the slip at full load. The slip at any load depends upon the watts lost in the resistance of the rotor winding.

It is evident that for any given number of bars in the rotor winding, the E.M.F. generated in the rotor winding will be proportional to its velocity relative to the field. Denote this velocity by v, and let the resistance of the rotor winding be ω, the watts lost in the winding will be proportional to $\frac{v^2}{\omega}$; now assume the resistance of the rotor to be increased, and vary the load on the motor in such a way that in spite of the altered resistance the current flowing in the rotor remains the same.

In order to put the same current through an increased resistance the E.M.F. must be increased, and this can only be obtained by a higher relative speed of field and rotor, the rotor will therefore pull up, and run with an increased slip. Continue the process until the rotor stands still; let Ω be the resistance of the rotor circuit necessary to obtain this result, and let V be synchronous speed. At stand-still, V is the relative speed of field and rotor, and the watts lost in rotor winding are now proportional to $\frac{V^2}{\Omega}$, and since the current remains unchanged $\frac{v}{\omega} = \frac{V}{\Omega}$ or $\Omega = \frac{V}{v}\omega$.

Again, $\frac{V^2}{\Omega}$ represents the total watts supplied to the rotor, because at standstill the rotor is doing no work, and the whole of the watts supplied to it must be wasted in its resistance, and since by hypothesis the current is not changed, $\frac{V^2}{\Omega}$, which represents the total watts supplied to the rotor at standstill, also represents the watts supplied to the rotor at full load. Therefore at full load $\frac{v^2}{\omega} \div \frac{V^2}{\Omega}$ represents the ratio of the watts lost in the rotor winding to the total watts supplied to the rotor. Substitute the value of Ω found above

$$\therefore \frac{v^2}{\omega} \div \frac{V^2}{\Omega} = \frac{v^2}{\omega} \times \frac{\frac{V}{v}\omega}{V^2} = \frac{v}{V}$$

but by definition $\frac{v}{V}$ is the slip, therefore the slip at full load is equal to the ratio of watts lost in the winding of the rotor to the total watts supplied to the rotor.

Evidently in order that the efficiency should be good, the loss in the rotor winding can only be a small percentage of the total watts supplied, and therefore the slip at normal load is always a small percentage of synchronous speed. Apart from any question of efficiency, the watts lost in the rotor must be kept down in value in order to prevent undue heating.

The rotor watts may be kept at about the same value as those lost in the stator winding, and these have been shown on page 243 to be 3,100. Say the watts lost in rotor resistance are 3,000, 100 H.P. = 74,600 watts, and the watts supplied to the rotor at full load are therefore

$74{,}600 + 3{,}000 = 77{,}600$, and the slip will be $\frac{3{,}000}{77{,}600} = \cdot 039$

or 3·9%. AG on the diagram (Fig. 117), therefore, represents a slip of 3·9%. At standstill the slip is 100%. On AG produced measure off AK to represent 100 to the same scale that AG represents 3·9, and join KF; this will cut the circle at a point M. The point M then represents standstill, OM is the current which the motor will take, and MN represents standstill torque. OM does not materially differ from OF, and the standstill current is frequently assumed to be equal to OF. To find the point M it is not actually necessary to draw AK, which is generally inconveniently long. Several geometrical devices depending on the properties of similar triangles are available for finding M without working to an inconvenient scale.

It is, of course, impossible to allow such a large current as that represented by OM to flow into the motor even temporarily, and means are taken, by employing some of the starting devices described in a later section, to cut down the starting current.

One way of doing this is to reduce the voltage on the motor terminals. The effect of decreasing the terminal E.M.F. is proportionately to reduce the scale of the diagram. If, for instance, the voltage of supply is reduced to one half, the current at any point of the circle will also be reduced to one half; that is, the diagram remains unaltered, but $\frac{1}{16}''$ instead of representing 10 amperes now represents 5 amperes only. The torque at any point, however, varies not as the voltage but as the square of the voltage. This is due to the fact that not only the current decreases proportionately to the voltage, but the field also decreases in the same proportion; the torque being the product of the current, the magnetic field varies not as the E.M.F. but as the E.M.F. squared.

The slip is measured to the same scale whatever the terminal voltage; it is the ratio of watts lost to total watts supplied to the rotor, and since these two quantities are of course altered in the same way by any change of E.M.F. the ratio between them remains unaltered.

§ 9. Rotor Winding.—A very great latitude of choice is possible in the rotor winding, especially if this is to be a squirrel cage. The number of bars is immaterial; any convenient number can be used. The only effect of varying the number of bars in the rotor winding is to alter the ratio of transformation between the primary and the secondary.

In considering the circle diagram it has been assumed that the ratio of transformation was unity, and that there was an equal number of bars on the stator and rotor; this is necessary in order to avoid great complication in interpreting the meaning of the different vectors, but in practice any ratio of transformation may be employed. If the rotor bars are few in number the E.M.F. generated will be small, but the current in each bar will have to be proportionately increased so that the ampere-turns per pole may have the required value.

In a squirrel-cage winding it is usual to arrange to have only one bar per slot, as this saves insulation in the slot and labour in winding.

The number of slots is generally not very far different from the number in the stator, and it is usual to choose a number of slots which shall be a prime number, so as to ensure that there shall be no submultiple common to the stator and rotor slots. This precaution is taken to avoid the effect which is known as cogging. A few words of explanation as to this effect are necessary.

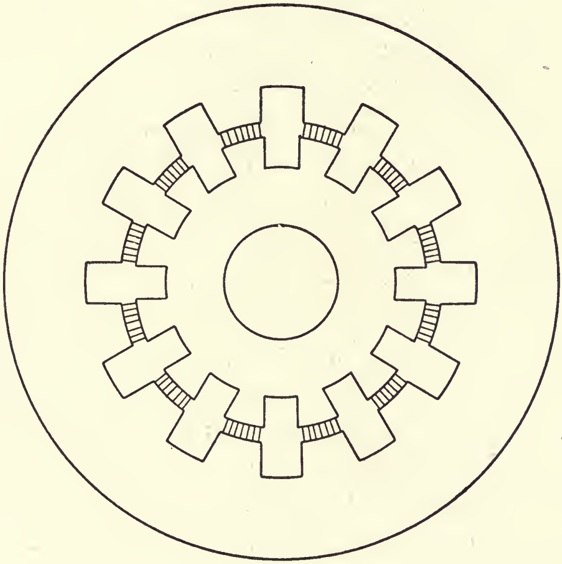


FIG. 118.

It is evident that if the stator and rotor had a very small number of slots, and the same number on rotor and stator as shown in Fig. 118, there would be some positions in which a stator tooth would be opposite to a rotor tooth at all parts of the circumference, and the magnetic flux passing across the air-gap from tooth to tooth would have a very strong tendency to keep the rotor in this position. There would thus be a strong force tending to oppose any motion; if, on the other hand, the teeth of one member happened to be opposite the slots of the other, this effect would not be present and starting would be much more easily effected. This is, of course, a very extreme case, but it can easily be seen by drawing one or two examples, that the same effect is present, though to a smaller degree, whenever the number of rotor slots and stator slots have a common submultiple. By making the number of rotor slots a prime number there can be no factor common to the number of slots in rotor and stator, and the cogging is reduced to a minimum.

The necessary size of the rotor bars is most easily arrived at from the consideration that the total current sheet round the whole rotor surface does not materially differ in value from the current sheet on the stator, and that it will therefore be necessary to have on the rotor a total section of copper not differing materially from that on the stator. As, however, on a squirrel cage the mean turn will be shorter than that of the stator winding (it usually works out at something like 80%), the section of rotor copper may be reduced to 80% of that on the stator, and still allow the watts lost in both members to remain the same.

On the stator of the 100 H.P. motor under consideration there are 540 bars, each having a section of ·046 square inch; the total section of copper is therefore $540 \times \cdot 046 = 25$ square inches, and there are on the stator 90 slots. It is proposed to put on the rotor 97 slots, each containing one bar, the total section of copper on the rotor to be $25 \times \cdot 8 = 20$ square inches; the section of each bar will be about $\frac{20}{90} = \cdot 22$ square inch. Say a bar $\cdot 6 \times \cdot 35$. As the insulation of the rotor is of comparatively little importance, the allowance in the slot can be kept quite low. A slot $\frac{3}{4}'' \times \cdot 45$ will be sufficient, the bare copper bar being threaded into the slot lined with an insulating trough consisting of a single thickness of presspahn, or similar material. The ends of all the bars at each end of the machine are then joined together by massive copper rings. The section of these rings must be sufficient to carry the current from half the bars under one pole; the other half flows in the opposite direction.

In the present case there are 97 bars and 6 poles, given say 16 bars per pole. It would therefore appear at first sight as if the section of the ring should be equal to that of 8 bars, but the maximum current will not be flowing at the same time in all the bars, and since the distribution of current round the rotor is approximately a sine curve, it will be sufficient if the section of the rings is $8 \times \cdot 21 \times \cdot 636 = 1\cdot 07$ square inch, ·636 being the factor by which the maximum value of a sine must be multiplied in order to give the mean value.

Had it been determined to have a wound rotor instead of a squirrel cage, it would have been necessary, in order to keep the three phases symmetrical, that the number of slots should be divisible by three, the number of phases, and by six, the number of poles. It must be noted that the cogging effect is not of so much importance in a wound rotor since the chief object of winding the rotor is to allow of the use of starting devices which will give a large starting torque. It is, however, not advisable that the number of slots on rotor and stator should be equal; thus, as there are on the stator five slots per pole per phase, the number on the rotor might be chosen to be either 4 or 6, giving 72 slots and 108 slots respectively. The number of bars on a wound rotor may be chosen simply for convenience in winding, and their section is found as in the case of the squirrel cage; the only limitation to the possible choice in the number of bars is that too high a number will give a high voltage per phase in the rotor winding at starting, whilst a very low number will give an excessive current to be carried by the slip rings and brushes.

§ **10. Starting Devices.**—An induction motor with a wound rotor is started by introducing resistance in the rotor circuit, and gradually cutting it out as the speed increases. The effect of this is not only to decrease the current taken by the motor, but also actually to increase the starting torque. Reference to Fig. 117 will make this clear. Under normal conditions the standstill point on the diagram is at M, the point M being found from the fact that the full-load slip is 3·9%, and this full-load slip is the ratio of watts lost in the rotor to total watts supplied to the rotor, in this case $\frac{3,000}{77,600}$ (see page 253).

If, however, the resistance of the rotor circuit is increased by introducing resistance between the slip rings in the proportion of 3·9 to 100, the watts lost in the rotor when full-load current is passing will be 77,600, and the slip will be $\frac{77,600}{77,600} = 1 = 100\%$; that is the point C will now represent standstill; and the motor will start with full-load torque and taking full-load current. As the speed increases, the resistance is gradually cut out, and finally the slip rings are short-circuited by a special switch on the rotor shaft; in some cases the brushes are at the same time lifted off the slip rings. These complications are evidently not desirable, and whenever it is possible to start against a comparatively small torque, a squirrel-cage rotor is preferred.

Motors of a small size with squirrel cage-rotors may be switched straight on to the mains, but as a general rule some device must be used to reduce the E.M.F. on the terminals at starting.

One way of doing this is to provide an auto-transformer which is switched in at starting, and reduces the E.M.F. on the stator terminals. After the motor has started the switch is thrown over to the running position when the auto-transformer is entirely disconnected and the motor terminals are connected directly to the line. An auto-transformer is practically the same thing as a static transformer; it, however, has only one winding per phase from which a tapping is taken, giving the reduced voltage required. Fig. 119 shows the connections of an auto-transformer consisting of three coils of wire wound on iron cores, and connected, star fashion, to the mains; tappings are taken from corresponding points of each coil to the motor. The E.M.F. on the motor terminals is then to the E.M.F. of the mains in the ratio of the turns in A B to the turns in A C.

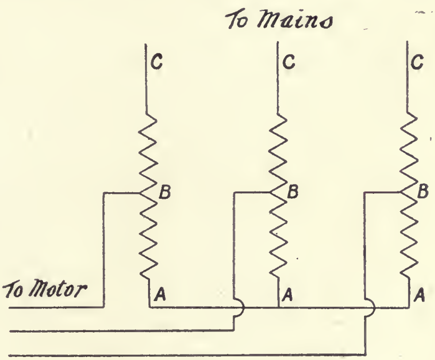


FIG. 119.

A numerical example will make the winding of the auto-transformer clear. Let each of the coils A C have two hundred turns, and let the portion A B have half of this, namely 100 turns. This apparatus will act exactly as a transformer having half the number of turns in the secondary which it has in the primary, that is, the E.M.F. between the tappings B will be half of the E.M.F. between the mains, and the current flowing in the tappings will be double the current taken from the mains. Referring again to the circle diagram in Fig. 117, the 100 H.P. motor if switched on to the mains at standstill, would take about 600 amperes, represented by O M, and would give about twice full-load torque, M N being about twice C D; the E.M.F. on each phase would be 290 volts.

If, however, the motor be connected to the tappings at B, (Fig. 119), the voltage on the terminals will be only $\frac{290}{2} = 145$, the current in each phase of the motor will be about 300, and since the current from the mains is only half of that in the tappings, it will be about 150 amperes, or one quarter of what would be taken by the motor if connected directly to the mains. On the other hand, the torque is also reduced to one quarter, for the E.M.F. on the motor terminals is half, and it has been seen that the torque varies as the square of the E.M.F. The motor will, therefore, take about $1\frac{1}{2}$ times full-load current from the mains, and give half full-load torque.

Another method of starting a squirrel-cage motor is to bring out the six ends of the three phases to a suitable switch, and connect them star for starting purposes, and throw them over to delta connections for running. Since the back E.M.F. of the star connection is 1·73 times that of the delta connection this is evidently equivalent, electrically, to using a transformer with a 1·73 ratio of transformation. The simplicity of this method is a strong recommendation, nothing being required except a simple switch to give the necessary change of connections. The disadvantages are that the motor must be designed for running on delta connections when in many cases a star connection would give a more convenient winding. Also when an auto-transformer is used several tappings giving different ratios of transformation can be taken off, and that one chosen for use which gives the best results. If the star-delta method of connection is used, the ratio must be 1 to 1·73, and no other choice can be made.

Many other more or less elaborate methods of starting

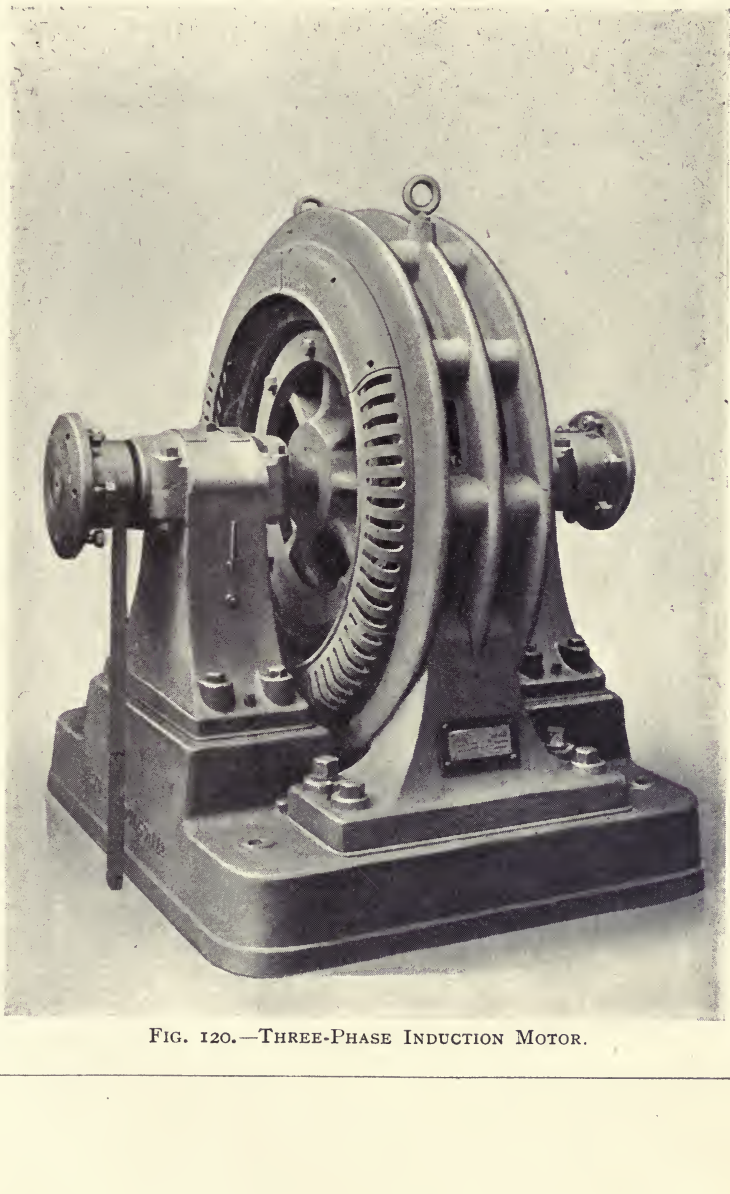


FIG. 120.—THREE-PHASE INDUCTION MOTOR.

squirrel-cage motors have been devised, but have been but little used in actual practice. A very full and interesting description of these methods will be found in Hobart's "Electric Motors."

A perspective view of a three-phase induction motor is given in Fig. 120.

CHAPTER X

OTHER VARIETIES OF ALTERNATING-CURRENT MOTORS

§ 1. Synchronous Motor, Single-Phase Induction Motor, and Single-Phase Commutator Motor.—Whilst the three-phase induction motor is in most common use at present, there are several other varieties of alternating-current motors on the market. The properties of these differ considerably from those of the induction motor, and in most cases render them less suitable for general use. The different classes of alternating-current motors to be noticed in this chapter are: (1) the synchronous motor, (2) the single-phase induction motor, and (3) various classes of motors which are closely related to one another and known as the single-phase commutator motors.

§ 2. The Synchronous Motor.—The synchronous motor consists merely of an alternating-current generator supplied with current and run as a motor. If supplied with current, a continuous-current machine will run as a motor, so the alternating-current machine if supplied with current will, when once up to synchronous speed, go on running and give a useful torque. It is, however, to be noticed that in this case it is necessary that the machine should be started by some external means, and should be running at or near synchronous speed before it is switched on to the mains.

When the machine is standing still, it gives no torque and has no tendency to run in either direction. The reason for this can easily be seen by reference to Fig. 121. At any given instant of time assume that the conductor on the armature marked A in the figure is opposite to a north pole of the magnet, and that it is carrying current in the direction from the front to back of the page. The conductor will then tend to move as shown by the arrow, but an instant later the direction of the current will have reversed and the torque and the tendency to motion in the armature will, therefore, also be reversed. The same argument holds for every conductor on the periphery of the armature, and thus with the armature standing still a series of impulses first in one direction and then in the other will be generated,

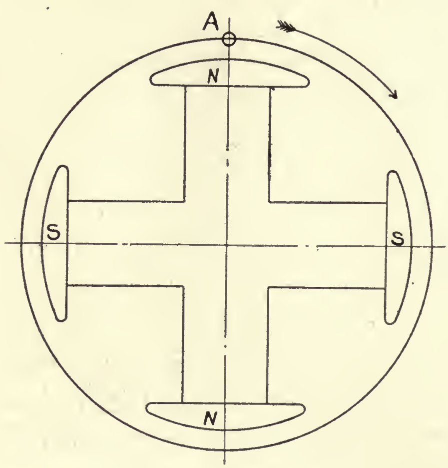


FIG. 121.

and the total effect will be *nil*. If, however, the armature is run up to synchronous speed, the case is quite different. Again, making the same assumptions as before, at any instant the wire A, carrying current from front to back of the page, will experience a torque in the direction shown by the arrow, and if the armature is running at synchronous speed it will, in an interval of time equal to one half period, have carried the conductor A through one pole pitch and have placed it in front of a south pole. During the same interval of time the current will have gone through a half period and will now be flowing from back to front of the page. Both the direction of the current and the polarity of the field are thus reversed, and the torque is therefore in the same direction; the armature will go on running at synchronous speed with the direction of rotation shown by the arrow. If the magnets rotate and the armature stands still the same argument holds, but for the opposite direction of rotation.

It is evident that a motor giving no torque at standstill, and requiring some external agency to get it up to speed, is not useful for general purposes, and this disability of the synchronous motor is further increased by the fact that it requires a supply of continuous current to excite its magnets.

To these two disadvantages is to be attributed the fact that synchronous motors have never come into general use for such work as driving factories and workshops.

There is, however, one class of work where the two drawbacks noticed above become of minor importance, and that is when the motors are to be used in sub-stations where alternating current has to be transformed into continuous current, usually at a lower pressure.

These stations are almost invariably parts of a large system of distribution, and the continuous-current bus-bars are constantly kept alive, that is, there is always a supply of continuous current available for exciting the synchronous motor. It is therefore not unusual in such stations to have some at least of the transformer sets, consisting of a continuous-current generator, coupled to a synchronous alternating-current motor. Such a set can always be started from the continuous-current end by temporarily using the continuous-current machine as a motor until synchronous speed is obtained. The synchronous machine is then connected to the alternating-current bars.

The fact that there is constant supply of continuous current available in these sub-stations is the circumstance which enables synchronous motors to be used to advantage for such work, and as a general statement it may be said that they are seldom used in any other way. Synchronous motors may be wound either for single-phase or for polyphase systems.

§ 3. The Single-Phase Induction Motor.—The single-phase induction motor resembles in every respect the polyphase induction motor already described in Chapter VIII., except that it is provided with one winding for single-phase current instead of two or three for multiphase currents. It is, however, in its properties generally very inferior to the polyphase machine.

It has been seen that the action of the polyphase induction motor can be ascribed to a rotating magnetic field due to the reaction of the different phase windings. The single-phase motor has no such rotating field, and as a consequence it gives no torque at starting, and must, like the synchronous motor, be provided with some external means for running it up to speed. This is usually done by providing a secondary winding displaced by 90 electrical degrees from the main winding; when it is desired to start the motor, this winding and the main winding are both connected to the mains, but there is introduced in the secondary winding either a capacity or a self-induction of such magnitude as considerably to alter the phase of the current flowing through the secondary as compared with that of the current flowing in the main winding. The result of this is that during the period of starting the motor acts very much as a two-phase motor.

If it were possible to introduce a sufficient amount of capacity or of self-induction to make the current in the one winding lag 90° behind the other, the motor would be truly a two-phase motor, but as this is of course impossible, the two currents do not differ by a quarter phase, and the starting torque is therefore not good; a large starting current is required, and only a small torque is obtained.

These are all considerable drawbacks, but it is, in addition, found that when the motor has run up to speed, and the secondary winding is cut out of action, it is even then seriously inferior, in respect of its power factor and of its capacity to stand overloads, to the corresponding polyphase machine. It is also inferior to this in weight efficiency, that is, the weight for a given rated output is high.

Notwithstanding these disadvantages the single-phase induction motor was, at any rate until recently, the only motor available for connection to the supply mains in those towns where a single-phase alternating-current system of distribution had been adopted, and in spite of all its inherent drawbacks, repeated and continuous efforts have been made to put a suitable single-phase induction motor on the market. Notwithstanding these efforts this motor can scarcely be said to have ever been a commercial success in Great Britain.

§ 4. **Single-Phase Commutator Motor.**—The single-phase commutator motors have been very largely worked at during the last few years. There are several kinds of them all closely related, and their development and the principles underlying their action may be looked at from several different points of view. Perhaps one of the simplest methods is to consider them as derived from the continuous-current series-wound motor. In such a motor the direction of rotation depends only on the internal connections of the machine—it depends only on the relative directions of the currents in the armature and in the magnet windings, and is independent of the actual direction of the current. If the direction of the current throughout the motor is reversed by altering its connections to the mains, the direction of rotation remains unaltered for, evidently, although the direction of the current in the armature conductors has been reversed, the polarity of the field has also been reversed, and the direction of rotation therefore remains the same as before; this is true however frequently the reversal of the current may take place. If a series-wound motor, then, constructed for use with continuous current is connected to alternating-current mains, it will rotate and rotate in the same direction as it would have done had it been connected to a continuous-current supply.

Such a motor would be very unsuitable for use with alternating currents, chiefly on account of the excessive eddy currents which would be generated in the solid magnets by the rapid reversals of the magnetic flux. This is easily remedied by laminating the magnet cores. The first of the A.C. commutator motors was built on these lines, and consisted of an ordinary series-wound continuous-current motor with the whole of its field magnet system laminated.

This motor was found still far from satisfactory, because the self-induction of the magnet and armature windings caused it to have a very poor power factor. It was then found that the inductance of the armature winding could by suitable construction be compensated for by the action of a coil displaced through 90 electrical degrees from the main magnet coil, carrying the main current, and a motor which is known as the compensated-series motor was thus evolved.

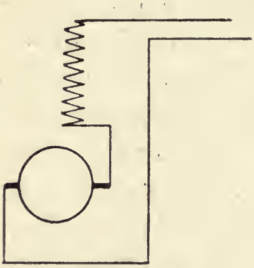


FIG. 122.

In order that the action of the compensating coil should not be complicated by the varying reluctance of the magnetic circuit, the construction of the motor was altered, and the magnet system instead of being similar to that of a continuous-current machine, having a certain number of polar projections carrying the magnet windings, was made similar to the stator of an induction motor. The compensated-series motor now consists of a continuous ring of laminated iron carrying the winding in slots on its internal periphery, and arranged so as to give the required number of poles. The compensating winding can then be wound in a second set of slots, arranged so as to be 90 electrical degrees apart from the main magnet winding. The arrangement of the different windings is shown in Figs. 122 and 123; Fig. 122 showing the series motor, and Fig. 123 the compensated-series motor. In these and the following figures relating to A.C. commutator motors the following conventions hold. The machine is in each case assumed to be two-pole, not because this construction is usually adopted (as a matter of fact these motors are almost invariably constructed with four or more poles), but because the simplicity of such a machine makes the diagram more easily understood, whilst the extension of the reasoning to any number of pairs of poles is easily followed.

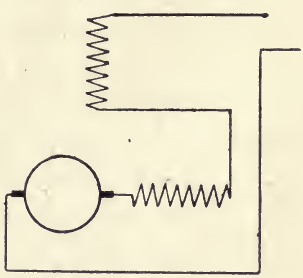


FIG. 123.

The windings are such that the direction of the magnetic flux coincides with the axis of the coil as shown on the diagram. The commutator is indicated by a circle with the brushes resting on its periphery, and it should be noted that the effect of the armature winding is always to give a magnetic field the direction of which coincides with the line joining the brushes. Thus, in Fig. 123, the main magnet winding, shown with its axis vertical, gives a field in a vertical direction; the field due to the armature current is along the horizontal, since the line joining the brushes is horizontal, and the compensating coil, with its axis horizontal, is wound so as to neutralise this field, and thus reduce the self-induction of the winding.

Although it is possible thus to neutralise the inductance of the armature, it is not possible by similar means to compensate for the inductance of the magnet coils. These are required to give the necessary magnetic field in which the armature conductors shall run, and it is evidently impossible that this should be neutralised.

It is, therefore, necessary in designing motors of this type that the number of turns in the magnet windings should be kept as low as possible, the number of armature conductors being correspondingly increased.

It has already been seen in the case of the induction motor that current may be induced in one winding by the action of the current flowing in another of the windings. Thus in the squirrel-cage motor the rotor winding is closed on itself, and is connected to no external source of supply; the current flowing in it is generated by magnetic induction from the stator winding.

Applying this principle to the motor in Fig. 123, the compensating coil may be short-circuited on itself, and made to carry current due simply to the armature flux. This modification is shown in Fig. 124. A further modification can be obtained by short-circuiting not the compensating coil, but the armature winding, giving the arrangement of circuits shown in Fig. 125. This is the first form of the motor known as the repulsion motor, and from the figure it is easily seen that the effect of the two coils, main and com-

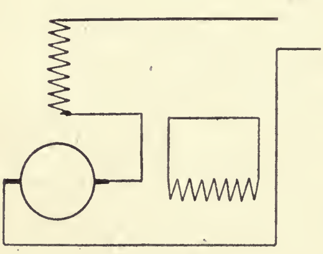


FIG. 124.

pensating, must be to give a resultant magnetic field, inclined at an angle between the two coils, the actual direction and intensity of which will depend on the relative number of turns in the two coils. It is evidently possible

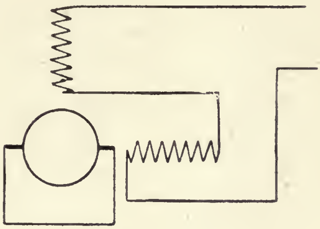


FIG. 125.

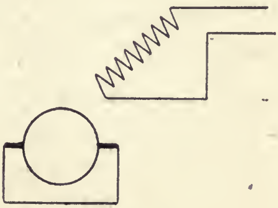


FIG. 126.

to substitute one coil for these two, so wound as to give the same resultant magnetic flux, and if this is done Fig. 126 will result.

Fig. 127 is identical with Fig. 126, except that it has been turned through an angle, so as to keep the axis of the coil in a vertical direction; it shows more clearly the usual way of looking at the repulsion motor, which is to consider it as being dependent for its action on the fact that the brushes are displaced through a certain angle.

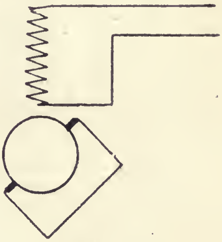


FIG. 127.

The addition of another pair of brushes to the commutator shown in Fig. 125 gives Fig. 128, in which the main field is due to the armature, not to the stator winding. A motor constructed on this principle is known as the compensated repulsion motor.

Of these different forms of the alternating-current commutator motors, the compensated-series motor as shown in Fig. 124, and the compensated-repulsion motor in Fig. 128 are both being developed, to a large extent, on the

Continent and in America for railway traction purposes. They have the advantage over continuous-current motors of allowing the use of static transformers for changing the pressure, so that the supply may be at a considerable pressure, thus saving copper in the line, and nevertheless be reduced to the ordinary working pressure at the motor. On the other hand, the motors being supplied with single-phase current only, the trolley line, and the apparatus for collecting current from it, are much simpler than would be the case if a three-phase induction motor were used. In addition to this these motors all have characteristics similar to that of the continuous-current series motor, that is, the speed decreases with the load. A motor behaving in this way has always been considered specially suitable for traction purposes, as any great increase in the load reduces the speed and therefore prevents the motor from being overloaded to the same extent as would be the case if it attempted to get over the obstacle with little or no reduction in speed.

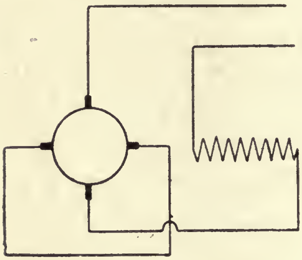


FIG. 128.

The characteristic of induction motors, on the other hand, is more similar to that of the shunt-wound continuous-current motor and they, therefore, do not have the same advantages when applied to traction purposes. It must, however, be noted that one great claim in favour of alternating-current systems has hitherto been the absence of the commutator and of commutator troubles. This class of motor has a commutator and is as liable as, in fact more liable than the continuous-current motor to give trouble on account of sparking at the brushes. It seems, nevertheless, to have a great future for long-distance traction, and in fact appears to some who have spent much time in developing it to have such good qualities that attempts have been and are still being made to produce an alternating-current commutator motor which shall have a characteristic more nearly resembling the shunt motor, and which will be easily capable of speed regulation, it being hoped that such a motor, if successful, might displace the continuous-current motor in general work, such as driving mills and workshops. Amongst such motors are the Creedy-Punga and the Fynn.

APPENDIX

DIMENSIONS, WEIGHTS, AND RESISTANCE OF COPPER WIRE

Round Wires

S.W.G.	*Diameter in inches*	*Area in sq. inches*	*Resistance per ft. at 60° F.*	*+ 20% allowance for heating and stretching*	*Weight in lbs. per ft.*
14	·082	·00503	·00162	·00195	·0194
15	·072	·00407	·00201	·00241	·0157
16	·064	·00322	·00254	·00304	·0124
17	·056	·00244	·00332	·00396	·0093
18	·048	·00181	·00451	·00541	·0070
19	·044	·00126	·00648	·00775	·0049
20	·036	·00102	·00801	·00970	·0039
21	·032	·00080	·0100	·0120	·0031
22	·028	·00061	·0133	·0159	·0024

Stranded Copper Conductors

7/13	·276	·046	·000174	·000185	·184
7/14	·24	·036	·000228	·000255	·137
7/15	·216	·029	·000280	·000315	·111
7/16	·192	·023	·000356	·000396	·088
7/17	·168	·018	·000455	·000509	·067
7/18	·144	·013	·000629	·000702	·049
7/19	·120	·0091	·00090	·00101	·034
7/20	·108	·0073	·00113	·00126	·021

RECTANGULAR CONDUCTORS

Dimensions in inches.	*Area in sq. inches*	*Resistance per foot at 60° F.*	*12 % allowance for heating.*	*Weight in lbs. per ft.*
·25 × ·04	·010	·00082	·00092	·0382
·25 × ·06	·015	·00054	·00061	·0572
·25 × ·08	·020	·00041	·00046	·0765
·3 × ·06	·018	·000454	·00051	·069
·3 × ·08	·024	·000341	·000381	·091
·3 × ·1	·03	·000274	·000306	·114
·3 × ·125	·0375	·000227	·000245	·144
·3 × ·15	·045	·000183	·000205	·172
·3 × ·175	·0525	·000157	·000176	·200
·3 × ·2	·06	·000137	·000153	·228
·5 × ·06	·03	·000273	·000306	·114
·5 × ·08	·04	·000206	·00023	·153
·5 × ·1	·05	·000165	·000184	·191
·5 × ·125	·0625	·000130	·000146	·240
·5 × ·15	·075	·000110	·000123	·286
·5 × ·175	·0875	·000094	·000105	·334
·5 × ·2	·100	·000082	·000092	·382
·6 × ·08	·048	·000171	·000191	·181
·6 × ·1	·06	·000137	·000153	·228
·6 × ·15	·09	·000092	·000102	·344
·6 × ·2	·12	·000064	·000072	·458
·6 × ·25	·15	·0000545	·000061	·575
·6 × ·3	·18	·0000454	·000051	·69
·6 × ·35	·21	·0000391	·0000441	·80
	1·00	·0000082	·0000092	3·82

The last line gives the resistance and weight of 1 ft. of copper one square inch in section. From this line the resistance and weight of any section not given in the table can be readily found, remembering that the resistance varies inversely and the weight directly as the cross section.

USEFUL CONSTANTS

One inch	= 2·54 centimeters.
One square inch	= 6·45 square centimeters.
One pound	= 455 grammes.
One kilogramme	= 2·2 lb.
One centimeter	= ·395 inch.
One square centimeter	= ·155 square inch.
746 watts	= One horse-power.
33,000 ft. lbs. per minute	= One horse-power.
44·2 ft. lbs. per minute	= One watt.
981 dynes	= One gramme.
447,000 dynes	= One pound.
1,000 amperes per square inch	= 155 amperes per square centimeter.
645 amperes per square inch	= 100 amperes per square centimeter.

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